The Cohomology Algebra of a Plane Curve and Related Topics

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Departamento de Matemáticas Universidad de Zaragoza

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Settings

$\mathcal{C}=\mathcal{C}_0\cup\mathcal{C}_1\cup...\cup\mathcal{C}_r\subset\mathbb{P}^2$

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 $H^*(X) = H^*(X; \mathbb{C})$

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 $H^*(X)=H^*(X;\mathbb{C})$

• Give a constructive description of $H^*(X)$ by generators and relations, as well as describe the product.

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Settings

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- Give a constructive description of H*(X) by generators and relations, as well as describe the product.
- Weak Combinatorial Invariants of C.
- Existence of an Orlik-Solomon-like Algebra.

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Settings

$$\mathcal{C} = \mathcal{C}_0 \cup \mathcal{C}_1 \cup ... \cup \mathcal{C}_r \subset \mathbb{P}^2$$

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- Give a constructive description of H^{*}(X) by generators and relations, as well as describe the product.
- Weak Combinatorial Invariants of C.
- Existence of an Orlik-Solomon-like Algebra.
- Formality.

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Definitions

 $\pi: S \rightarrow \mathbb{P}^2$

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Definitions

$$\begin{array}{ccccc} \pi : & \mathcal{S} & \to & \mathbb{P}^2 \\ & \cup & & \cup \\ & \bar{\mathcal{C}} & \to & \mathcal{C} \end{array}$$

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Definitions

Definition

The sheaf $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{\mathcal{C}})$ is the sheaf of *log-resolution logarithmic forms* of \mathcal{C} w.r.t. π .

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Remark

• The sheaf $\pi_* \mathcal{E}^*_S(\log \overline{\mathcal{C}})$ is independent of the resolution.

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The sheaf $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{C})$ is the sheaf of *log-resolution logarithmic forms* of C w.r.t. π .

Remark

- The sheaf $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{\mathcal{C}})$ is independent of the resolution.
- Denote it by $\mathcal{E}^*_{\mathbb{P}^2}(\log \mathcal{C})$.

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Definitions

$$\begin{array}{cccc} \pi : & \mathcal{S} & \to & \mathbb{P}^2 \\ & \cup & & \cup \\ & \bar{\mathcal{C}} & \to & \mathcal{C} \end{array}$$

Definition

The sheaf $\pi_* \mathcal{E}^*_{\mathcal{S}}(\log \overline{C})$ is the sheaf of *log-resolution logarithmic forms* of C w.r.t. π .

Remark

- The sheaf $\pi_* \mathcal{E}^*_S(\log \overline{C})$ is independent of the resolution.
- Denote it by E^{*}_{ℙ²}(log C).
- *E*^{*}_{ℙ²}(log *C*) inherits a weight filtration *W*_{*}.

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$H^{i}(S; W_{i}\mathcal{E}^{*}_{S}(\log \overline{C}))$

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 $\begin{array}{c} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \underset{\text{Leray}}{\otimes} \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) \end{array}$

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$\begin{array}{c} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \underset{\text{Leray}}{\otimes} \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) \\ & \underset{\text{H}^{i}(X)}{\otimes} \end{array}$

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$$\begin{array}{ll} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \underset{\mathbb{L} \text{ teray}}{\otimes} \\ H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) & \rightarrow & H^{i}(S; W_{i}/W_{i-1}) \\ & \underset{\mathbb{Q} \text{ DeRham}}{\otimes} \\ & H^{i}(X) \end{array}$$

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$$\begin{array}{ll} H^{i}(\mathbb{P}^{2}; \mathcal{W}_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \underset{\text{Leray}}{\otimes} \text{ Leray} \\ H^{i}(S; \mathcal{W}_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) & \rightarrow & H^{i}(S; \mathcal{W}_{i}/\mathcal{W}_{i-1}) & \simeq & H^{0}(\bar{\mathcal{C}}^{[i]}) \\ & \underset{\text{H}^{i}(X)}{\otimes} \end{array}$$

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$$\begin{array}{ll} H^{i}(\mathbb{P}^{2}; \ensuremath{\mathcal{W}_{\mathbb{P}^{2}}}(\log \mathcal{C})) \\ & & \gtrless_{\mathsf{Leray}} \\ H^{i}(S; \ensuremath{\mathcal{W}_{\mathcal{S}}}(\log \bar{\mathcal{C}})) & \to & H^{i}(S; \ensuremath{\mathcal{W}_{i-1}}) & \simeq & H^{0}(\bar{\mathcal{C}}^{[i]}) \\ & & \gtrless_{\mathsf{DeRham}} \\ & & H^{i}(X) \end{array}$$

Such a residue map will be denoted by Res^[/].

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$$\begin{array}{ll} H^{i}(\mathbb{P}^{2}; W_{i}\mathcal{E}_{\mathbb{P}^{2}}^{*}(\log \mathcal{C})) \\ & \underset{\mathbb{L}^{\operatorname{tray}}}{\otimes} & H^{i}(S; W_{i}\mathcal{E}_{S}^{*}(\log \bar{\mathcal{C}})) & \rightarrow & H^{i}(S; W_{i}/W_{i-1}) & \simeq & H^{0}(\bar{\mathcal{C}}^{[i]}) \\ & \underset{\mathbb{H}^{i}(X)}{\otimes} & H^{i}(X) \end{array}$$

Such a residue map will be denoted by Res^[/].

In more generality:

$$H^{i}(\mathbb{P}^{2}; W_{k}\mathcal{E}^{*}_{\mathbb{P}^{2}}(\log \mathcal{C})) \xrightarrow{\mathsf{Res}^{[i,k]}} H^{i-k}(\bar{\mathcal{C}}^{[k]}).$$

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Theorem (-,D.Matei)

Under the above conditions:

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Theorem (-,D.Matei)

Under the above conditions: Res^[1,1] is injective.

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Theorem (-,D.Matei)

Under the above conditions:

Res^[1,1] is injective.

Solution If $\psi \in \mathcal{E}^2(\mathbb{P}^2)(\log \mathcal{C})$ is such that $\operatorname{Res}^{[2,2]} \psi = 0$ and $\operatorname{Res}^{[2,1]} \psi = 0$, then $\psi = 0$.

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Example

Consider $f = y^2 - x^4$, $C = \{f = 0\}$, and the 2-form $\frac{dx \wedge dy}{f}$.

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Example

Consider $f = y^2 - x^4$, $C = \{f = 0\}$, and the 2-form $\frac{dx \wedge dy}{f}$.

$$\frac{dx \wedge dy}{f} \stackrel{x=u_1}{\xleftarrow{y=u_1v_1}} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1^2)}$$

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$$\frac{dx \wedge dy}{f} \stackrel{\stackrel{x = u_1}{\longleftarrow} u_1}{\leftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1^2)} \stackrel{\stackrel{u_1 = u_2v_2}{\longleftarrow} u_2 \times dv_2}{\leftarrow} \frac{du_2 \wedge dv_2}{u_2v_2^2(1 - u_2^2)}$$

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which is *not* logarithmic.

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which is not logarithmic.

However, if $\psi = \varphi \frac{dx \wedge dy}{f}$, then

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Log-resolution Logarithmic Forms Poincaré Residue Operators

Example

Consider
$$f = y^2 - x^4$$
, $C = \{f = 0\}$, and the 2-form $\frac{dx \wedge dy}{f}$.

$$\frac{dx \wedge dy}{f} \stackrel{\stackrel{x = u_1}{\longleftarrow} u_1 \vee u_1}{\leftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1^2)} \stackrel{\stackrel{u_1 = u_2v_2}{\longleftarrow} u_2v_2^2(1 - u_2^2)}{\leftarrow}$$

which is *not* logarithmic.

However, if $\psi = \varphi \frac{dx \wedge dy}{f}$, then • $\varphi \in (x, y) \Rightarrow \psi \in \mathcal{E}_0^2(\log \mathcal{C}).$

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Example

Consider
$$f = y^2 - x^4$$
, $C = \{f = 0\}$, and the 2-form $\frac{dx \wedge dy}{f}$.

$$\frac{dx \wedge dy}{f} \stackrel{\stackrel{x = u_1}{\longleftarrow} u_1 \vee u_1}{\longleftarrow} \frac{du_1 \wedge dv_1}{u_1(v_1^2 - u_1^2)} \stackrel{\stackrel{u_1 = u_2 v_2}{\longleftarrow} u_2 v_2^2(1 - u_2^2)}{\underbrace{du_2 \wedge dv_2}{\longleftarrow} \frac{du_2 \wedge dv_2}{u_2 v_2^2(1 - u_2^2)}$$

which is not logarithmic.

However, if $\psi = \varphi \frac{dx \wedge dy}{f}$, then • $\varphi \in (x, y) \Rightarrow \psi \in \mathcal{E}_0^2(\log \mathcal{C}).$ • Moreover, if $\varphi \in (y) \Rightarrow \left(\operatorname{Res}^{[2,2]} \psi\right)_P = 0$ at all $P \in \overline{\mathcal{C}}^{[1]}$ infinitely near 0.

A Presentation of H* (X) Weak Combinatorics

Theorem

The following is a presentation of $H^*(X)$:

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• Generators in degree 1: σ_i , i = 1, ..., r,

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The following is a presentation of $H^*(X)$:

- Generators in degree 1: σ_i , i = 1, ..., r,
- Generators in degree 2:

$$\begin{array}{ll} \psi_{\mathcal{P}}^{\delta_1,\delta_2}, & \mathcal{P} \in \mathcal{C}_i \cap \mathcal{C}_j, \delta_1 \in \Delta_{\mathcal{P}}(\mathcal{C}_i), \delta_2 \in \Delta_{\mathcal{P}}(\mathcal{C}_j) \\ \psi_{\infty}^{i,k_i}, & i = 1, ..., r, k_i = 1, ..., d_i - 1 \\ \eta^{i,s_i}, \bar{\eta}^{i,s_i}, & i = 1, ..., r, s_i = 1, ..., g_i. \end{array}$$

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• Relations:

$$\begin{split} \psi_{P}^{\delta_{1},\delta_{2}} &= -\psi_{P}^{\delta_{2},\delta_{1}} \\ \psi_{P}^{\delta_{1},\delta_{2}} + \psi_{P}^{\delta_{2},\delta_{3}} + \psi_{P}^{\delta_{3},\delta_{1}} = \mathbf{0} \end{split}$$

for any $P \in C_i \cap C_j \cap C_k$ and $\delta_1 \in \Delta_P(C_i), \delta_2 \in \Delta_P(C_j), \delta_3 \in \Delta_P(C_k)$.

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Theorem

The following is a presentation of $H^*(X)$:

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Relations:

$$\begin{split} \psi_{P}^{\delta_{1},\delta_{2}} &= -\psi_{P}^{\delta_{2},\delta_{1}} \\ \psi_{P}^{\delta_{1},\delta_{2}} + \psi_{P}^{\delta_{2},\delta_{3}} + \psi_{P}^{\delta_{3},\delta_{1}} = \mathbf{0} \end{split}$$

for any $P \in C_i \cap C_j \cap C_k$ and $\delta_1 \in \Delta_P(C_i), \delta_2 \in \Delta_P(C_j), \delta_3 \in \Delta_P(C_k)$. • *Product:*

$$\sigma_i \wedge \sigma_j = \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_1, \delta_2) \psi_P^{\delta_1, \delta_2} + d_i \sum_{k_j=1}^{d_j-1} \psi_{\infty}^{j, k_j} - d_j \sum_{k_j=1}^{d_j-1} \psi_{\infty}^{i, k_j}.$$

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A Presentation of H* (X) Weak Combinatorics

Remark

Note that from the given presentation one can deduce that $H^*(X)$ only depends on the following invariants of C:

$$(\{1,...,r\}, \mathcal{S} = \operatorname{Sing} \mathcal{C}, \{\Delta_P\}_{P \in \mathcal{S}}, \{\phi_P\}_{P \in \mathcal{S}}, \{\mu_P\}_{P \in \mathcal{S}})$$

such an ordered set of invariants of C will be referred to as the *Weak Combinatorics of* C.

A Presentation of H* (X) Weak Combinatorics

Remark

Note that from the given presentation one can deduce that $H^*(X)$ only depends on the following invariants of C:

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such an ordered set of invariants of C will be referred to as the *Weak Combinatorics of* C.

Hence

Theorem

The cohomology algebra of X only depends on the weak combinatorics of C and the genera of its irreducible components.

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Generalized Orlik-Solomon Algebra

Consider $\omega \in H^1(X)$.

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Generalized Orlik-Solomon Algebra

Consider $\omega \in H^1(X)$.

$$0 \to H^1(X) \stackrel{\bullet \wedge \omega}{\longrightarrow} H^2(X) \to 0 \qquad (H^*(X), \wedge \omega)$$

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Generalized Orlik-Solomon Algebra

Consider $\omega \in H^1(X)$.

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Definition

The *i-th Resonance Variety of X* is defined as

$$\mathcal{R}^i(X) := \{\omega \in H^1(X) \mid h^1(H^*(X), \wedge \omega) \geq i\}$$

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Generalized Orlik-Solomon Algebra

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Definition

The *i-th Resonance Variety of X* is defined as

$$\mathcal{R}^i(X) := \{\omega \in H^1(X) \mid h^1(H^*(X), \wedge \omega) \ge i\}$$

Remark

Note that for any graded algebra A^* one can analogously define the *i*-th Resonance Variety $\mathcal{R}^i(A)$ of A^* .

Generalized Orlik-Solomon Algebra

Theorem

There is a purely combinatorial Orlik-Solomon-like graded algebra A^* whose resonance varieties are isomorphic to $\mathcal{R}^i(X)$.

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Generalized Orlik-Solomon Algebra

Theorem

There is a purely combinatorial Orlik-Solomon-like graded algebra A^* whose resonance varieties are isomorphic to $\mathcal{R}^i(X)$.

$$A^{1} := \sum_{i=1}^{r} \sigma_{i} \mathbb{C} \quad A^{2} := \sum_{P \in \mathcal{S}} \frac{\bigwedge^{2} A_{P}}{l_{P}},$$

where

$${\mathcal A}_{\mathcal P}:=\sum_{\delta\in\Delta_{\mathcal P}}\psi^\delta_{\mathcal P}\mathbb C$$

$$\mathbf{I}_{\mathbf{P}} := \langle \psi_{\mathbf{P}}^{\delta_1} \wedge \psi_{\mathbf{P}}^{\delta_2} + \psi_{\mathbf{P}}^{\delta_2} \wedge \psi_{\mathbf{P}}^{\delta_3} + \psi_{\mathbf{P}}^{\delta_3} \wedge \psi_{\mathbf{P}}^{\delta_1} \rangle_{\mathbb{C}}$$

and

$$\sigma_i \wedge \sigma_j := \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_1, \delta_2) \psi_P^{\delta_1, \delta_2}$$

Remark

Note that $A^2 \ncong H^2(X)$.

Generalized Orlik-Solomon Algebra

Corollary

The resonance varieties $\mathcal{R}^i(X)$ of the complement of a plane curve \mathcal{C} are determined by the combinatorics of \mathcal{C} .

Generalized Orlik-Solomon Algebra

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The resonance varieties $\mathcal{R}^i(X)$ of the complement of a plane curve \mathcal{C} are determined by the combinatorics of \mathcal{C} .

Definition (J.Hilman, C.Sabbah, Esnault-Schechtman-Viehweg, M.Falk, D.Arapura)

The *i*-th *Characteristic Variety* $Char^{i}(X)$ of a topological space X is

 $\{\rho \in \operatorname{Hom}(\pi_1(X), \mathbb{C}^*) \mid h^1(X; \mathbb{C}_{\rho}) \geq i\}$

Generalized Orlik-Solomon Algebra

Corollary

The resonance varieties $\mathcal{R}^i(X)$ of the complement of a plane curve \mathcal{C} are determined by the combinatorics of \mathcal{C} .

Definition (J.Hilman, C.Sabbah, Esnault-Schechtman-Viehweg, M.Falk, D.Arapura)

The *i*-th Characteristic Variety Charⁱ(X) of a topological space X is

 $\{\rho \in \operatorname{Hom}(\pi_1(X), \mathbb{C}^*) \mid h^1(X; \mathbb{C}_{\rho}) \geq i\}$

Theorem (A.Libgober, D.Cohen-A.Suciu, E.Hironaka)

If X is the complement of a projective curve $\mathcal{R}^{i}(X)$ is the tangent cone of $\operatorname{Char}^{i}(X)$ at the origin **1**.

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Corollary

The components of $\operatorname{Char}^{i}(X)$ passing through **1** are combinatorially determined.

Max-Noether Fundamental Theorem Revisited

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Definition (P.Deligne-P.Griffiths-J.Morgan-D.Sullivan)

A differential space X is called *formal* if its algebra of global differential forms $(\mathcal{E}(X), d)$ is formal.

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Max-Noether Fundamental Theorem Revisited

Theorem (- D.Matei, A.D.Macinic)

The complement of a plane curve X is a formal space.

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Max-Noether Fundamental Theorem Revisited

Theorem (- D.Matei, A.D.Macinic)

The complement of a plane curve X is a formal space.

• $(\mathcal{E}(X), d) \stackrel{q.i.}{\simeq} (\mathcal{E}(\mathbb{P}^2)(\log \mathcal{C}), d),$ • $(H(\mathcal{E}(X)), 0) \stackrel{q.i.}{\simeq} (H(X), 0),$

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 $H^*(X) \stackrel{e}{\rightarrow} \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C})$

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$$\begin{array}{rccc} H^*(X) & \stackrel{e}{\to} & \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C}) \\ [\sigma_i] & \mapsto & \sigma_i \\ [\psi_{\mathcal{P}}^{\delta_1, \delta_2}] & \mapsto & \psi_{\mathcal{P}}^{\delta_1, \delta_2} \\ [\psi_{\infty}^{i,k_i}] & \mapsto & \psi_{\infty}^{i,k_i} \\ [\eta_{\infty}^{i,s_i}] & \mapsto & \eta^{i,s_i} \end{array}$$

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$$\begin{array}{rccc} H^*(X) & \stackrel{e}{\to} & \mathcal{E}^*(\mathbb{P}^2)(\log \mathcal{C}) \\ [\sigma_i] & \mapsto & \sigma_i \\ [\psi_{\mathcal{P}}^{\delta_1, \delta_2}] & \mapsto & \psi_{\mathcal{P}}^{\delta_1, \delta_2} \\ [\psi_{\infty}^{i, k_i}] & \mapsto & \psi_{\infty}^{i, k_i} \\ [\eta_{\infty}^{i, s_i}] & \mapsto & \eta^{i, s_i} \end{array}$$

Can we choose forms so that e is well-defined?

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Max-Noether Fundamental Theorem Revisited

$$\psi_P^{\delta_1,\delta_2} + \psi_P^{\delta_2,\delta_3} + \psi_P^{\delta_3,\delta_1} = \mathbf{0}$$

Choose δ_P at each $P \in S$, then

$$\psi_{\textit{P}}^{\delta_1,\delta_2}=\psi_{\textit{P}}^{\delta_{\textit{P}},\delta_2}-\psi_{\textit{P}}^{\delta_{\textit{P}},\delta_1}$$

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Max-Noether Fundamental Theorem Revisited

$$\sigma_i \wedge \sigma_j =$$

$$= \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_i, \delta_j) \psi_P^{\delta_i, \delta_j} +$$

$$+ d_i \sum_{k_j=1}^{d_j-1} \psi_{\infty}^{j, k_j} - d_j \sum_{k_j=1}^{d_j-1} \psi_{\infty}^{i, k_j}.$$

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Max-Noether Fundamental Theorem Revisited

$$\begin{aligned} \sigma_i \wedge \sigma_j &= \\ &= \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_j, \mathcal{C}_i) \psi_P^{\delta_P, \delta_j} - \sum_{P \in \mathcal{C}_i \cap \mathcal{C}_j} \mu_P(\delta_i, \mathcal{C}_j) \psi_P^{\delta_P, \delta_i} + \\ &+ d_i \sum_{k_j = 1}^{d_j - 1} \psi_{\infty}^{j, k_j} - d_j \sum_{k_i = 1}^{d_i - 1} \psi_{\infty}^{i, k_i}. \end{aligned}$$

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Let C_i, C_j, C_k be such that:

• $\mu_P(\delta_k, C_i) = \mu_P(\delta_k, C_j),$

then

$$\sigma_i \wedge \sigma_j + \sigma_j \wedge \sigma_k + \sigma_k \wedge \sigma_i = 0 \tag{1}$$

Note that if $C_k = \alpha C_i + \beta C_i$, then (1) is trivial.

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Max-Noether Fundamental Theorem Revisited

Theorem (Max-Noether Fundamental Theorem (M.Noether,..., W.Fulton))

Let *F*, *G*, and *H* be three plane curves with no common components. If $H_P \in (F_P, G_P)$ at any $P \in V(F) \cap V(G)$, then there exist two forms $A, B \in \mathbb{C}[x, y, z]$ such that

H = AF + BG

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Remark

The conditions $H_P \in (F_P, G_P)$ at any $P \in V(F) \cap V(G)$ are commonly known as the *Noether Conditions*.

Max-Noether Fundamental Theorem Revisited

Definition

Three curves F, G, and H satisfying (\bigcirc) are said to belong to a *combinatorial pencil*.

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Max-Noether Fundamental Theorem Revisited

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Three curves F, G, and H satisfying (\bigcirc) are said to belong to a *combinatorial pencil*.

Theorem (-,M.A.Marco)

If F, G, and H belong to a primitive combinatorial pencil, then they belong to an algebraic pencil ($H = \alpha F + \beta G$).

Max-Noether Fundamental Theorem Revisited

Remark

• The Noether Conditions can be replaced by the Combinatorial Conditions.

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Max-Noether Fundamental Theorem Revisited

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Proposition

Any combinatorial pencil admits a primitive refinement.

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Max-Noether Fundamental Theorem Revisited

Remark

- The Noether Conditions can be replaced by the Combinatorial Conditions.
- Primitive translates into a minimality condition.

Proposition

Any combinatorial pencil admits a primitive refinement.

This proves the formality of X.

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Open Problems

• Are there also *nice* combinatorial descriptions of *H*^{*}(*X*) in higher dimensions?

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Open Problems

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- Are the complements of hypersurfaces in the projective space formal?

Open Problems

- Are there also *nice* combinatorial descriptions of *H*^{*}(*X*) in higher dimensions?
- Are the complements of hypersurfaces in the projective space formal?
- What about toric varieties, or weighted projective spaces?