

Arrangements de droites: problèmes élémentaires qui deviennent complexes

Enrique ARTAL BARTOLO

Departamento de Matemáticas-IUMA
Facultad de Ciencias

Séminaire de Mathématiques et leurs Applications
Pau, 15 octobre 2026

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Line arrangements: elementary problems become complex

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Lines in the plane

Properties

- ▶ Given two distinct points in the plane, there is a unique line passing through them



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- ▶ We state that all the extra points form a line, **the line at infinity**



Lines in the plane

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Modern version

$\mathbb{K}$ a field, V $\mathbb{K}$ -vector space of dimension 3:

- ▶ $\mathbb{P}(V)$: space of 1-dimensional vector subspaces of V .
- ▶ Lines of $\mathbb{P}(V)$: 2-dimensional vector subspaces of V .
- ▶ Standard linear algebra arguments show the above properties.

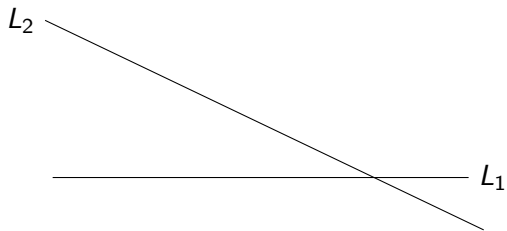


Arrangements of lines

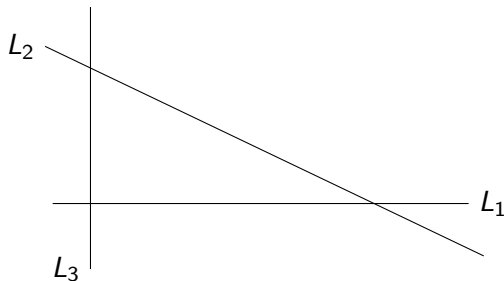
————— L_1



Arrangements of lines



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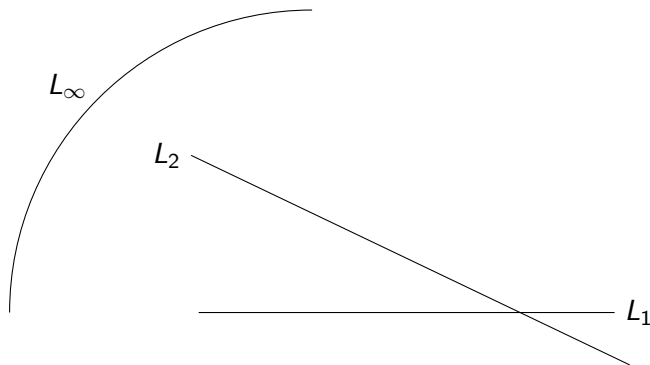


Combinatorics

$\mathcal{C}_1$: 3 lines intersecting at distinct points



Arrangements of lines

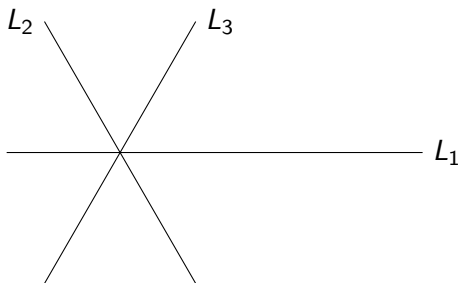


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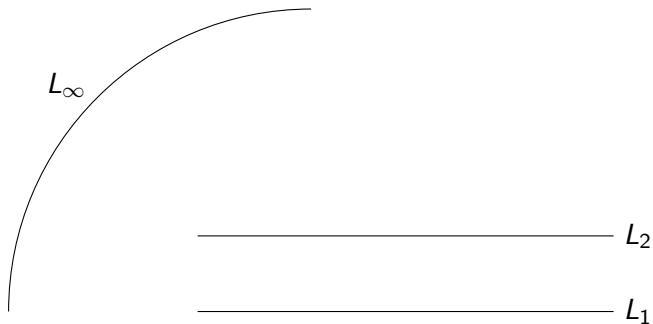
Arrangements of lines



Combinatorics
 $\mathcal{C}_2$: 3 concurrent lines



Arrangements of lines



Combinatorics
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Combinatorics (linear algebra without equations)

Definition

A **combinatorics** $\mathcal{C}$ consists of:

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Number of combinatorics

- ▶ 0 lines: 0 combinatorics



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Number of combinatorics

- ▶ 1 lines: 1 combinatorics



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Number of combinatorics

- ▶ 3 lines: 2 combinatorics



Combinatorics (linear algebra without equations)

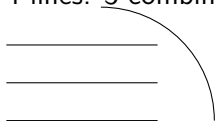
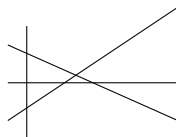
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Number of combinatorics

- ▶ 4 lines: 3 combinatorics



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Number of combinatorics

- ▶ 5 lines: 5 combinatorics



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Number of combinatorics

- ▶ 6 lines: 10 combinatorics



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Number of combinatorics

- ▶ 7 lines: 24 combinatorics



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Number of combinatorics

- ▶ 8 lines: 69 combinatorics
- ▶ 9 lines: 384 combinatorics
- ▶ 10 lines: 5250 combinatorics
- ▶ 11 lines: 232 929 combinatorics (89MB)
- ▶ 12 lines: 28 872 973 combinatorics (9,8GB)



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<https://matroid.mathematik.uni-siegen.de/index.html>



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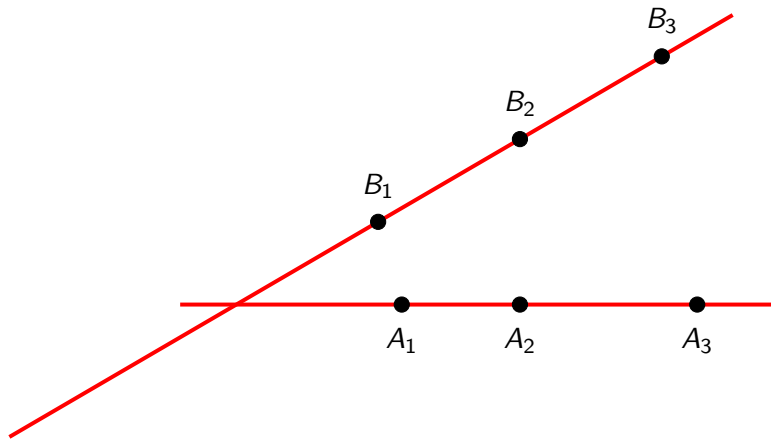
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Question

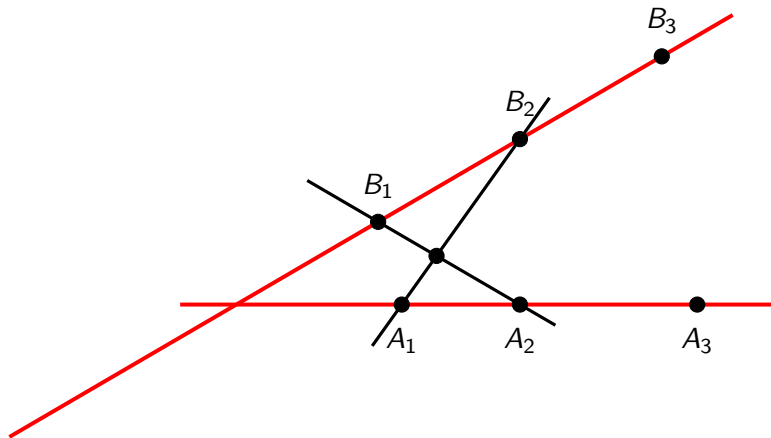
Can we realize all patterns?



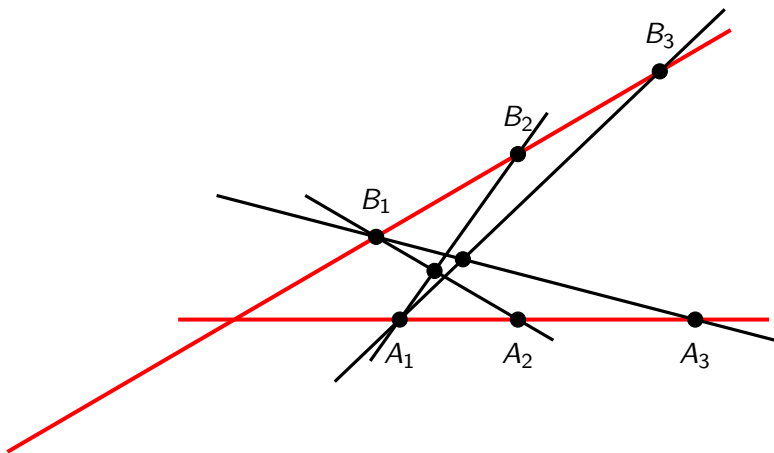
Pappus Theorem



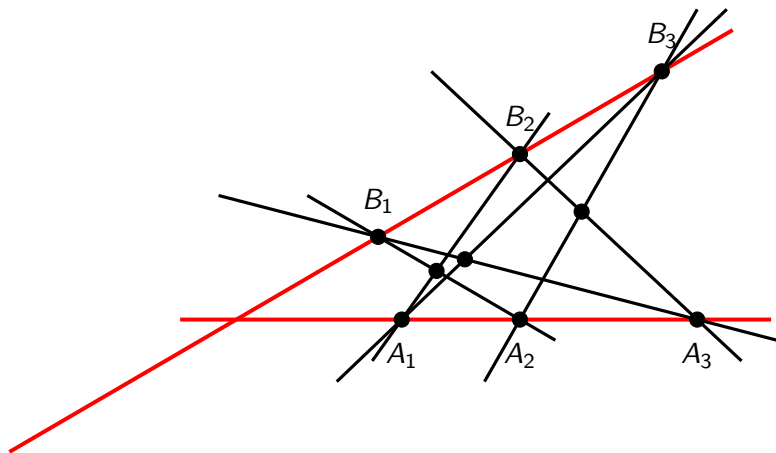
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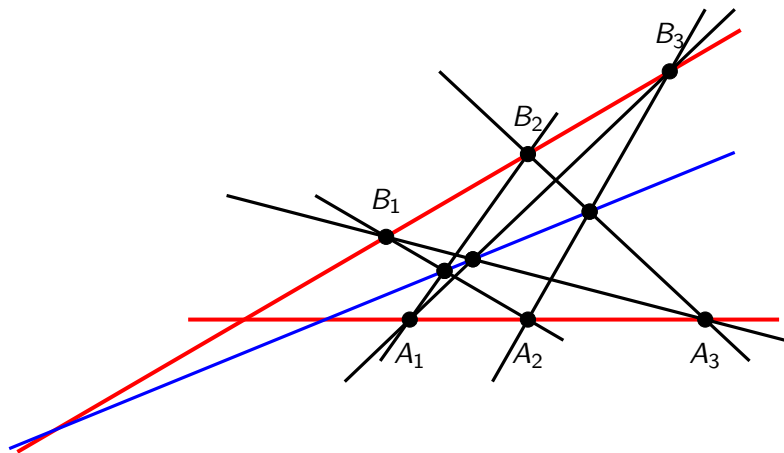
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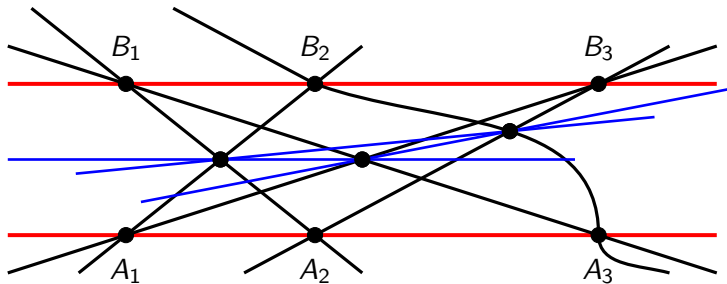
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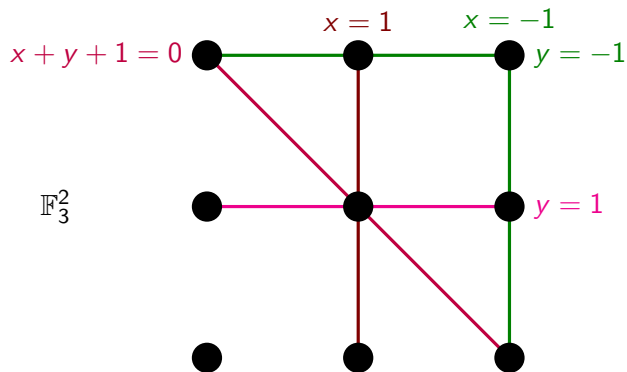
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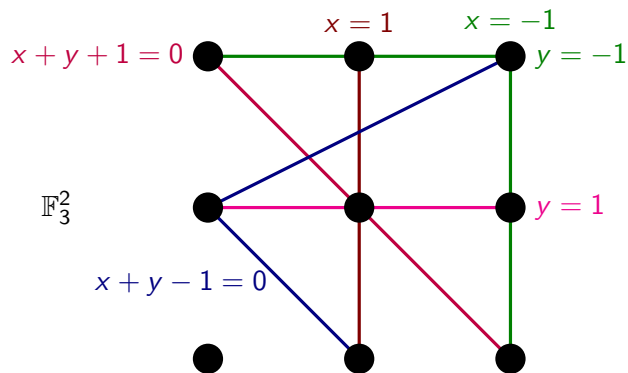
Non-realizable combinatorics



MacLane combinatorics

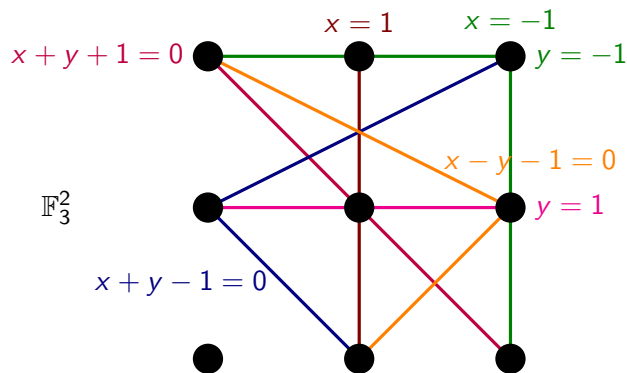


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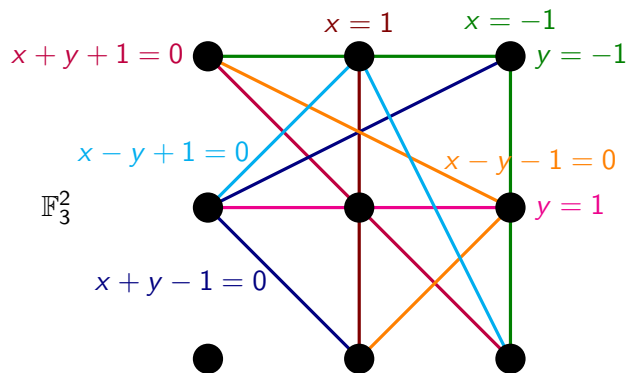


Only intersection at the nodes

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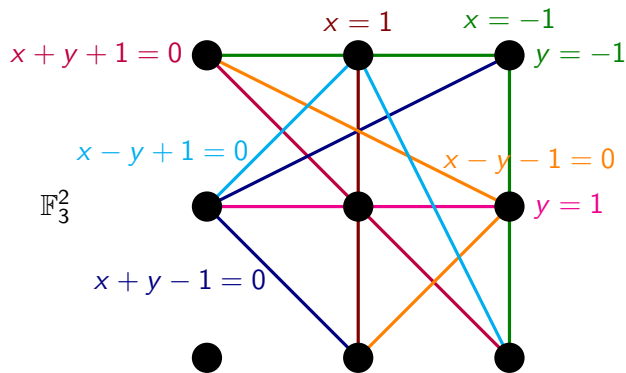


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Smallest combinatorics realizable in $\mathbb{C}$ and not in $\mathbb{R}$.

OK for $\mathbb{F}_3$

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- ▶ Combinatorics for which $\mathbb{Q}$ is not a definition field, but a number field (and then $\mathbb{C}$) is: topology, Galois theory, algebraic geometry



Algebraic questions

Terao's conjecture



Algebraic questions

Terao's conjecture

- The module of algebraic derivations in $\mathbb{C}^{n+1}$ is the free $S := \mathbb{C}[x_0, \dots, x_n]$ -module:

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- ▶ $\mathcal{A}$ arrangement of linear hyperplanes in $\mathbb{C}^{n+1}$ define by linear forms $\alpha_H = 0$, $H \in \mathcal{A}$. The module of logarithmic derivations is
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- ▶ **Solution for $n = 3$ up to 12 lines by Faenzi and Vallès.**



Topology of complex line arrangements

Main problem



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- Determine conditions for the existence of a homeomorphism $h : (\mathbb{P}^2, \mathcal{A}_1) \rightarrow (\mathbb{P}^2, \mathcal{A}_2)$, $\mathcal{A}_1, \mathcal{A}_2$ line arrangements.



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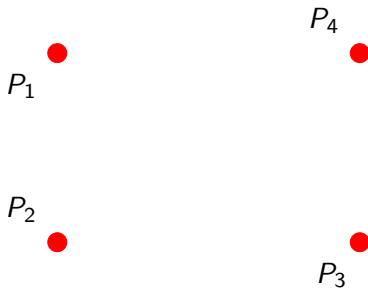
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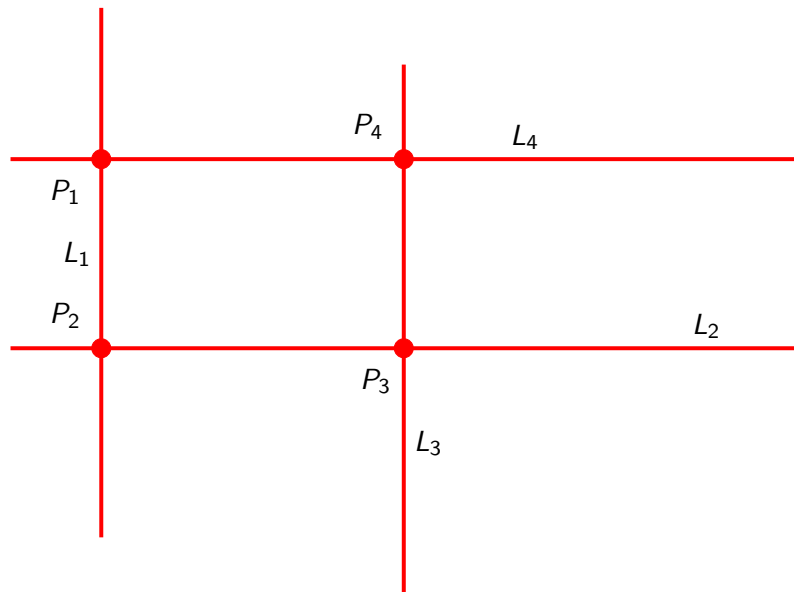
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- ▶ Rybnikov, A., Cogulludo, Carmona, Marco, Florens, Guerville, Viu, Rodau, Manouras



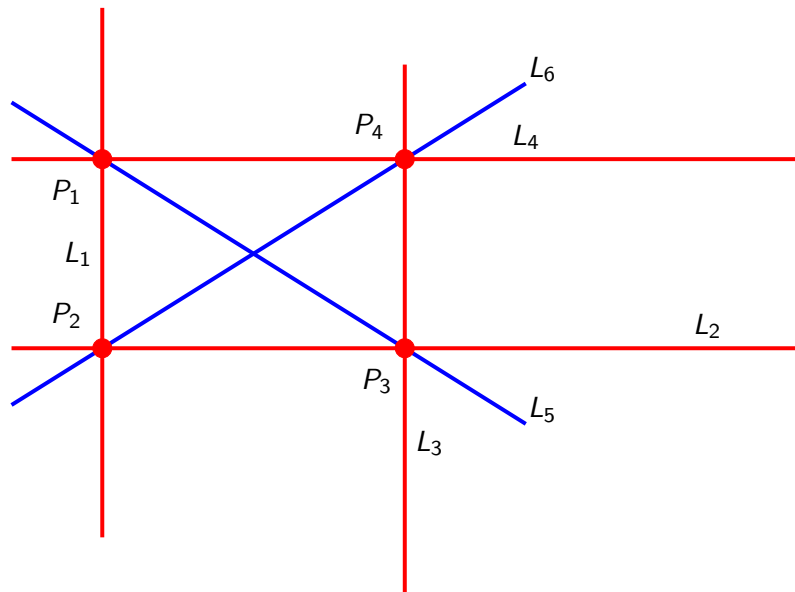
Combinatorics born in Pau



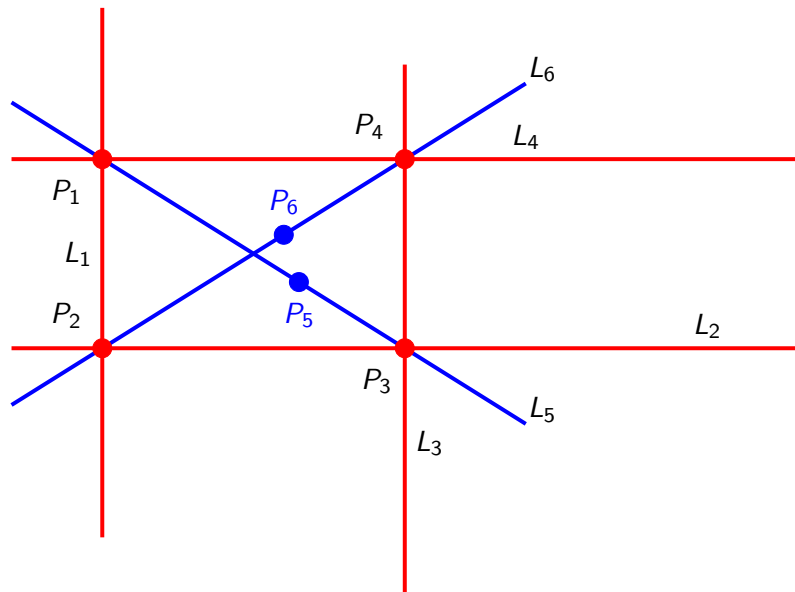
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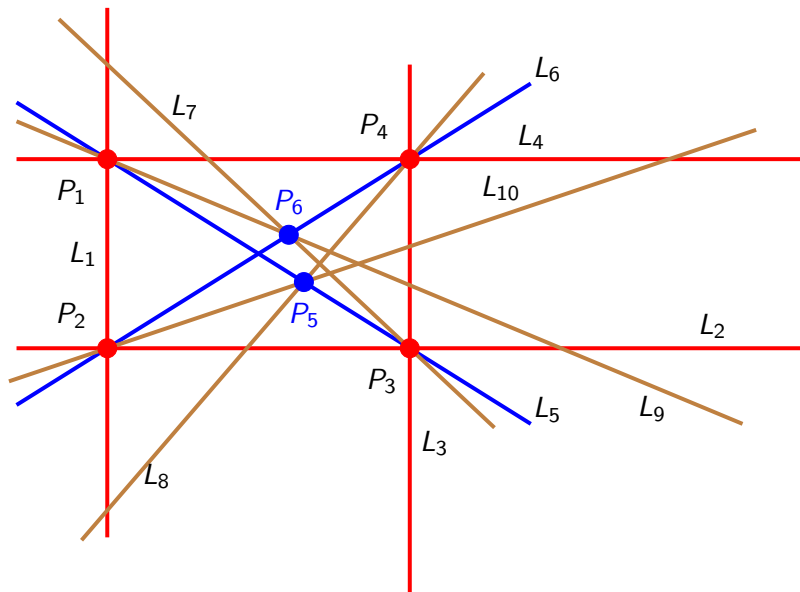
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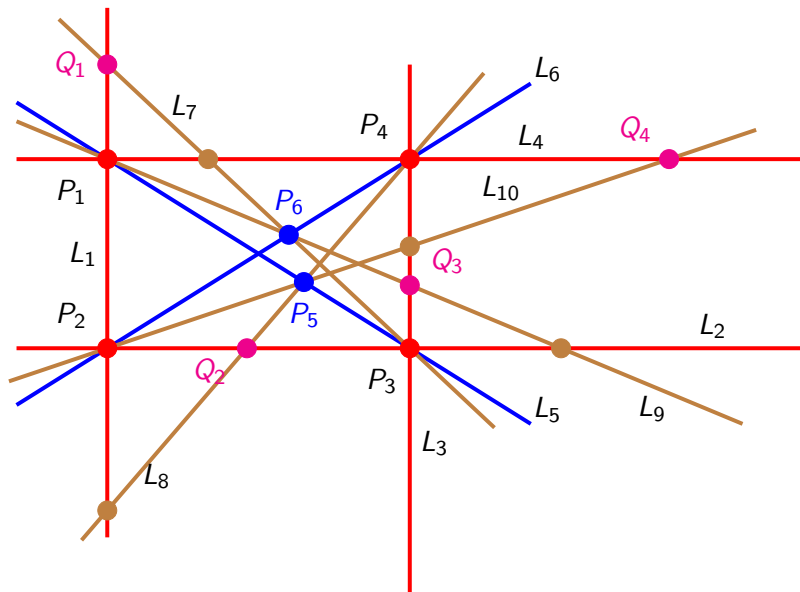
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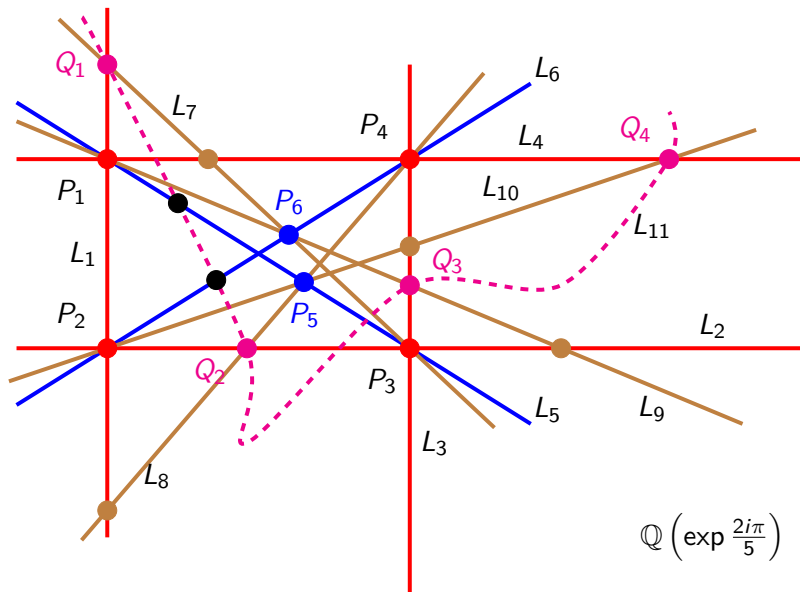
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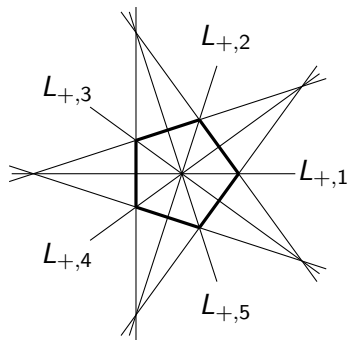
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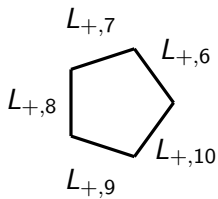
Combinatorics born in Pau



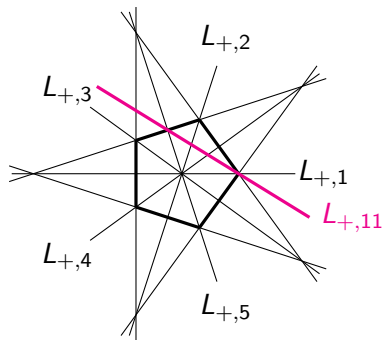
Pentagon and pentagram



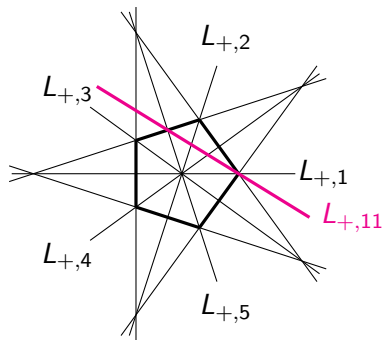
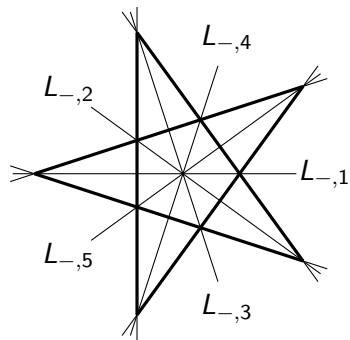
Pentagon and pentagram



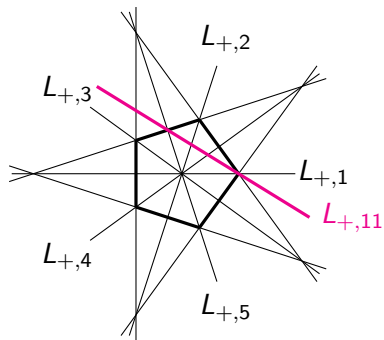
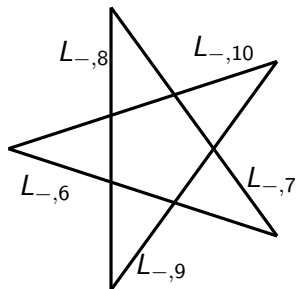
Pentagon and pentagram



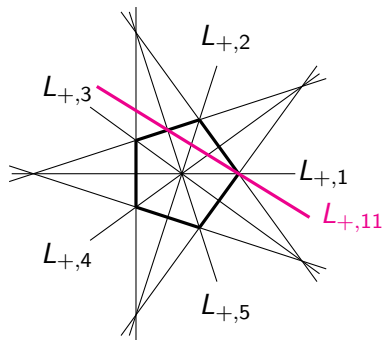
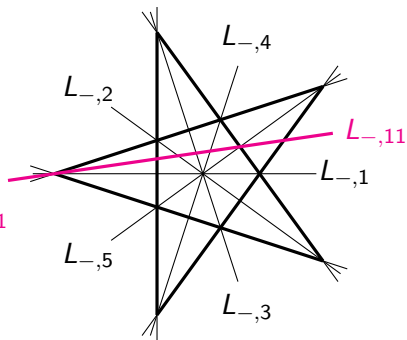
Pentagon and pentagram



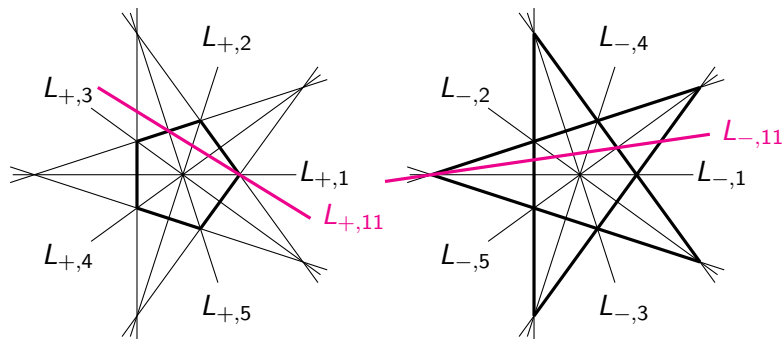
Pentagon and pentagram



Pentagon and pentagram



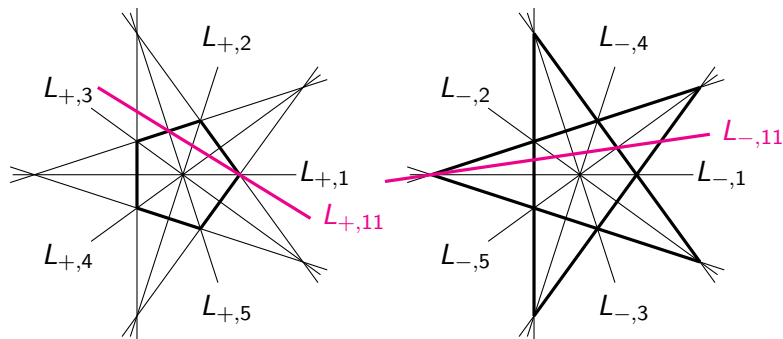
Pentagon and pentagram



- $\mathbb{Q}[\sqrt{5}]$ is a (minimal) definition field for this combinatorics

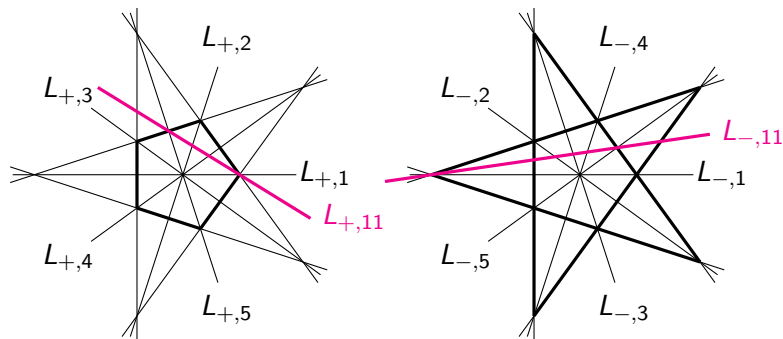


Pentagon and pentagram



- ▶ $\mathbb{Q}[\sqrt{5}]$ is a (minimal) definition field for this combinatorics
- ▶ $L_{i,+} \mapsto L_{i,-}$ via $\sqrt{5} \mapsto -\sqrt{5}$.

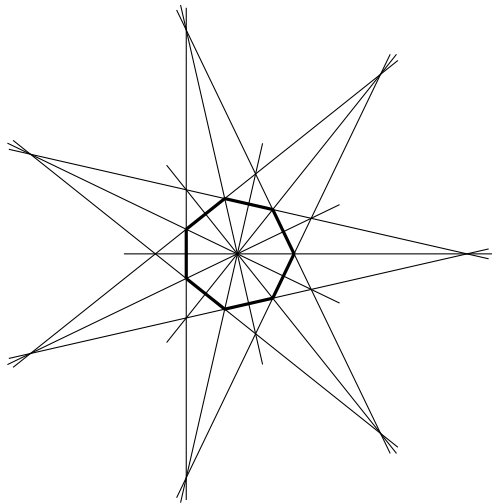
Pentagon and pentagram



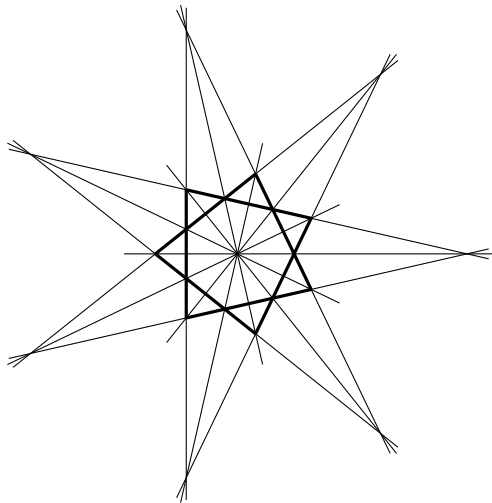
- ▶ $\mathbb{Q}[\sqrt{5}]$ is a (minimal) definition field for this combinatorics
- ▶ $L_{i,+} \mapsto L_{i,-}$ via $\sqrt{5} \mapsto -\sqrt{5}$.
- ▶ In $\mathbb{CP}^2$ it is not possible to deform one realization into the second one (A_-Carmona-Cogolludo-Marco)



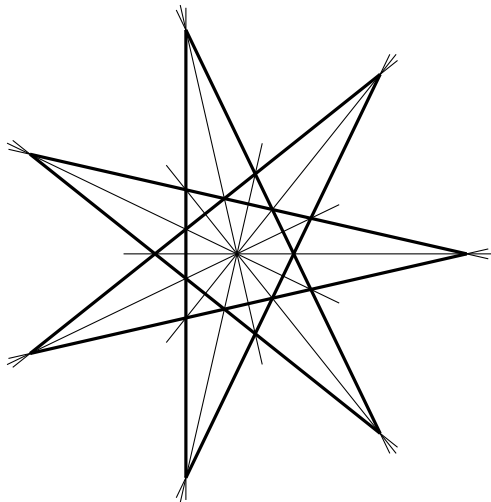
More regular polygons



More regular polygons

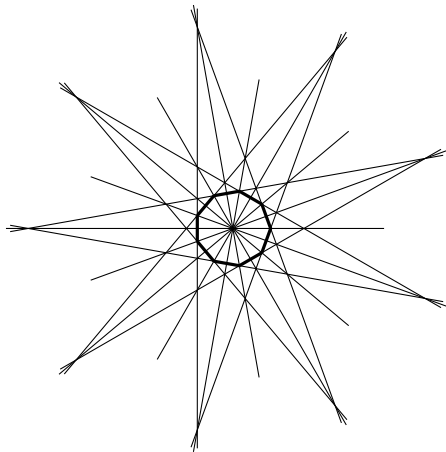


More regular polygons

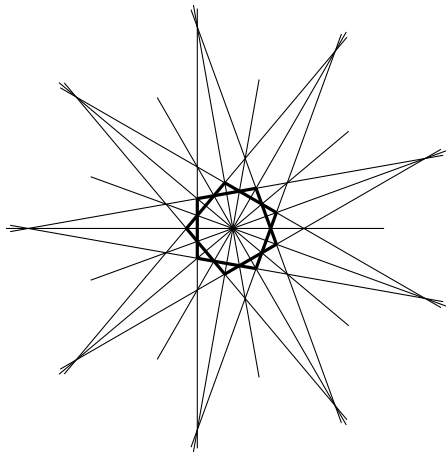


$\mathbb{Q}\left[2\cos\frac{2\pi}{7}\right]$ with minimal polynomial $t^3 + t^2 - 2t - 1$

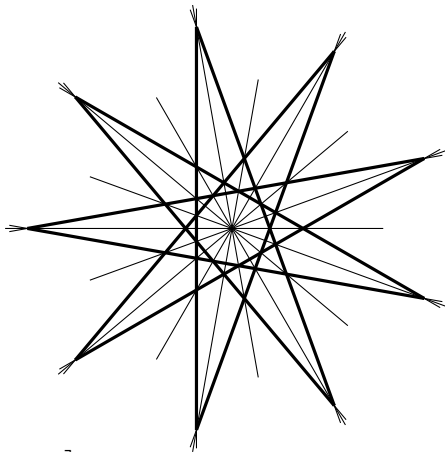
And more



And more

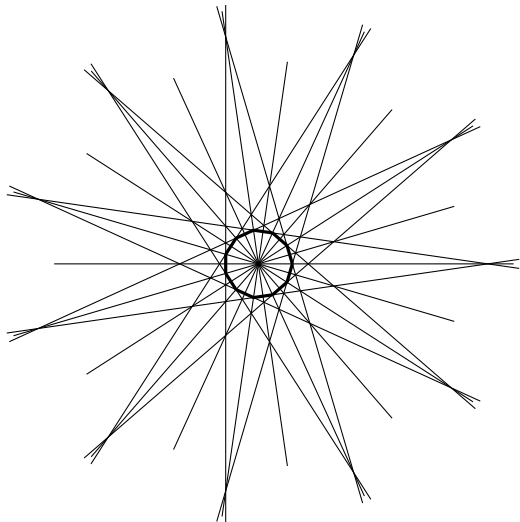


And more

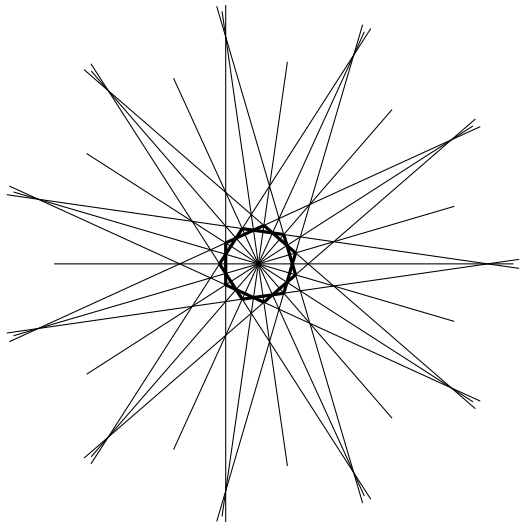


$\mathbb{Q} \left[2 \cos \frac{2\pi}{9} \right]$ with minimal polynomial $t^3 - 3t + 1$

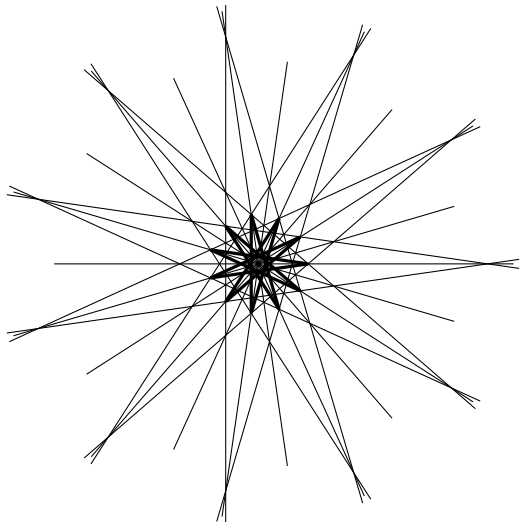
11 is enough



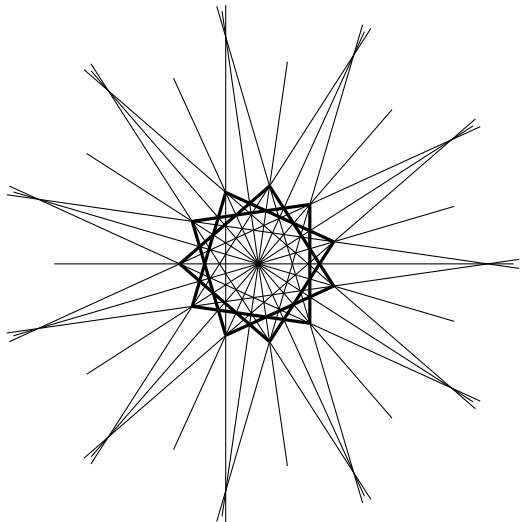
11 is enough



11 is enough

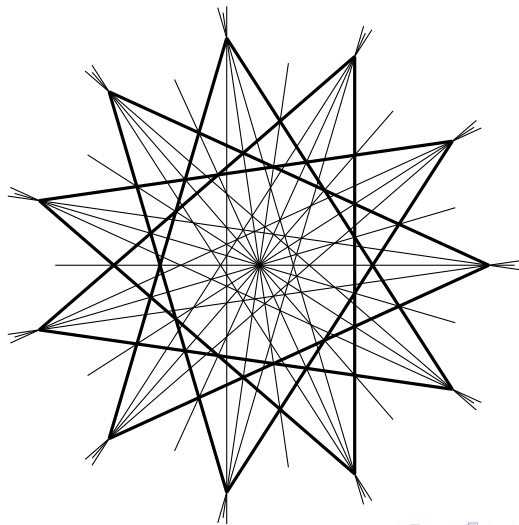


11 is enough



11 is enough

$\mathbb{Q}\left[2\cos\frac{2\pi}{11}\right]$ with minimal polynomial $t^5 + t^4 - 4t^3 - 3t^2 + 3t + 1$



Merci de votre attention!

