

Falsos productos de rectas proyectivas y cubiertas cíclicas.

Enrique ARTAL BARTOLO

Departamento de Matemáticas
Facultad de Ciencias
Instituto Universitario de Matemáticas y sus Aplicaciones
Universidad de Zaragoza

Reunión conjunta RSME-SMM
Guanajuato, 17 de Junio de 2021

Trabajo conjunto con J.I. Cogolludo y J. Martín Morales



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- ▶ $D \sim dH$ puede tener varias soluciones.



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► $\gamma_1, \dots, \gamma_r \in \mathbb{C} \subset \mathbb{P}^1$

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- ▶ **Riemann-Hurwitz:** $g(C) = 1 + d \frac{r-2}{2} - \frac{1}{2} \sum_{i=1}^r \gcd(d, m_i).$

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$$H^1(X; \mathbb{C}) \cong H^1(X; \mathcal{O}_X) \oplus H^0(X; \Omega_X^1), \quad H^1(X; \mathcal{O}_X) \cong \overline{H^0(X; \Omega_X^1)}.$$



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Singularidades de hipersuperficie

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 - ▶ **Determina la Estructura de Hodge Mixta (y acción de la monodromía);**



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$$\begin{array}{ccccc} (Z, E_Z) & \xrightarrow{\nu} & (Y, E_Y) & \longrightarrow & (X, E) \\ & & \downarrow & & \downarrow \pi \circ F \\ & & (\mathbb{C}^{n+1}, 0) & \longrightarrow & (\mathbb{C}^{n+1}, 0) \\ & & t & \longmapsto & t^e \end{array}$$

- ▶ $E_X \cong E_Y$, $E_Z \rightarrow E_X$ cubiertas cíclicas de las componentes.
- ▶ Sucesión espectral de Steenbrink:
 - ▶ Determina la Estructura de Hodge Mixta (y acción de la monodromía);
 - ▶ **Degenera en E_2 .**



Singularidades de hipersuperficie

- ▶ $(V, 0) \subset (\mathbb{C}^{n+1}, 0)$, sing. aislada de hipersuperficie, $V = F^{-1}(0)$.
- ▶ $\pi : (X, E) \rightarrow (\mathbb{C}^{n+1}, 0)$ morfismo propio, resolución encajada.
- ▶ $D = \pi^*(V) = \sum_{i \in I} m_i D_i$ divisor con cruces normales simples.
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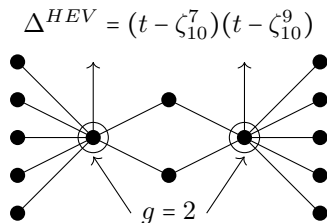
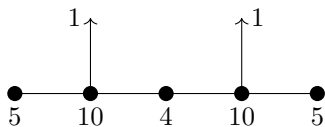
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 - ▶ En E_1 : cohomologías de componentes de E_Z .

Ejemplos de normalización semiestable: curvas

Ejemplo

A'Campo: $(y^2 - x^3)(y^3 - x^2) = 0$.



Ejemplos de normalización semiestable: superficies

Ejemplo (Singularidades superaisladas)

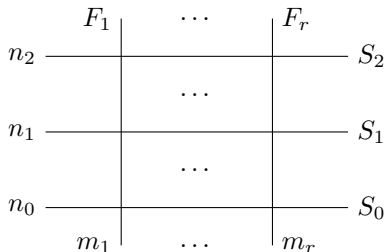
$$f_d + f_{d+1} + \cdots = 0 \quad \text{Sing}_{\mathbb{P}}(f_d = 0) \cap (f_{d+1} = 0)_{\mathbb{P}} = \emptyset$$



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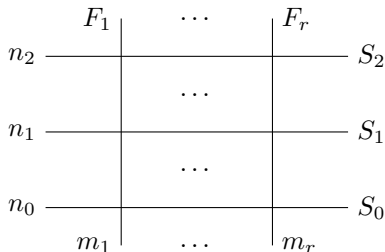
► $D = n_0 S_0 + n_1 S_1 + n_2 S_2 + \sum_{j=1}^r m_j F_j.$



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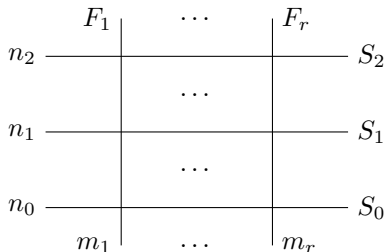
► $D = n_0 S_0 + n_1 S_1 + n_2 S_2 + \sum_{j=1}^r m_j F_j.$

► $dn = n_0 + n_1 + n_2, \quad dm = \sum_{j=1}^r m_j.$

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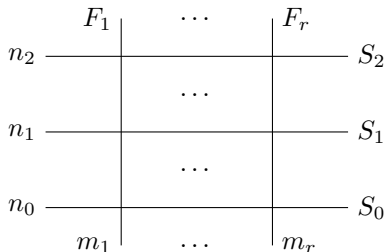
$$\blacktriangleright H = nS + mF.$$



Ejemplos de normalización semiestable: superficies

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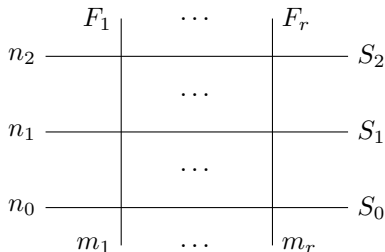


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- ▶ $dn = n_0 + n_1 + n_2$, $dm = \sum_{j=1}^r m_j$.
- ▶ $H = nS + mF$.
- ▶ $L^{(l)} \sim \left(-ln + \left\lfloor \frac{ln_0}{d} \right\rfloor + \left\lfloor \frac{ln_1}{d} \right\rfloor + \left\lfloor \frac{ln_2}{d} \right\rfloor \right) S + \left(-lm + \sum_{i=1}^r \left\lfloor \frac{lm_i}{d} \right\rfloor \right) F$

Ejemplos de normalización semiestable: superficies

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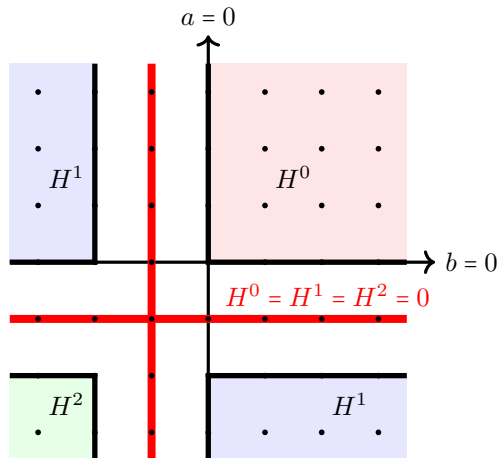
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Cohomología de $\mathbb{P}^1 \times \mathbb{P}^1$

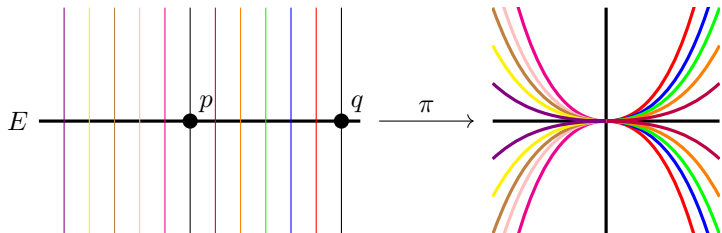


$$\chi(\mathcal{O}_{\mathbb{P}^1 \times \mathbb{P}^1}(aS + bF)) = (a + 1)(b + 1)$$



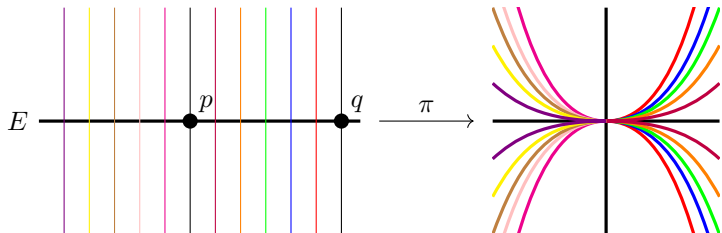
Explosiones ponderadas y \mathbb{Q} -resoluciones

Explosiones ponderadas

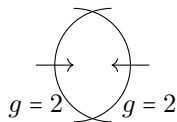
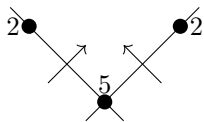


Explosiones ponderadas y \mathbb{Q} -resoluciones

Explosiones ponderadas



Ejemplo de A'Campo



Explosiones ponderadas y \mathbf{Q} -resoluciones

Singularidades de Yomdin: $f_d + f_{d+k} + \cdots = 0$,

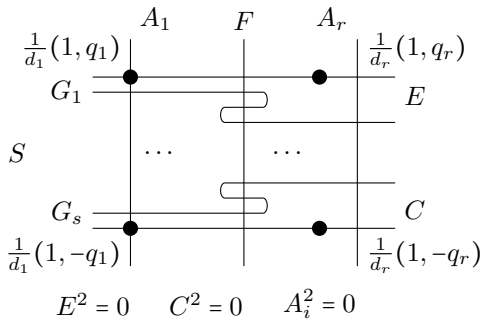
$$\text{Sing}_{\mathbb{P}}(f_d = 0) \cap (f_{d+k} = 0)_{\mathbb{P}} = \emptyset$$



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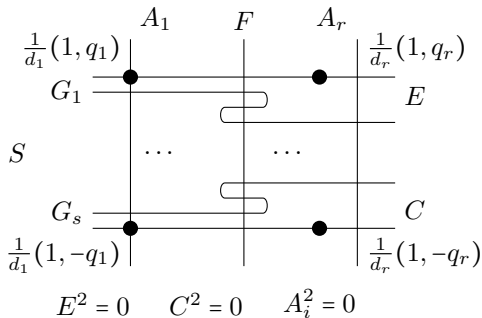
$$d_i A_i \sim F$$

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$$d_i A_i \sim F$$

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- ▶ $\text{Cl}(H) = ?$
- ▶ $H^*(S; \mathcal{O}_S(D)) = ?$



Esnault-Viehweg con singularidades cociente

Teorema (A-Cogolludo-Martín)

- ▶ X superficie proyectiva con singularidades cocientes cíclicas,
- ▶ $\pi : Y \rightarrow X$ cubierta cíclica (d, D, H) , $D \sim dH$
- ▶ D cruces \mathbb{Q} -normales simples,
- ▶ $\sigma : Y \rightarrow Y$ generador de $\text{Aut } \pi$, $\zeta_d = \exp \frac{2i\pi}{d}$
- ▶ $H_l \subset H^1(Y; \mathcal{O}_Y)$ subespacio ζ_d^l -propio de σ^* .

$$H_l \cong H^1(X; \mathcal{O}(L^{(l)})), \quad L^{(l)} = -lH + \sum_{i \in I} \left\lfloor \frac{lm_i}{d} \right\rfloor D_i.$$



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Invariantes numéricos

$$L^{(l)} \cdot B = - \sum_{i \in I} \left\{ \frac{lm_i}{d} \right\} (D_i \cdot B)$$



Índice

Falso $\mathbb{P}^1 \times \mathbb{P}^1$ asociado a una cubierta cíclica de \mathbb{P}^1

Cubierta cíclica de \mathbb{P}^1

- ▶ $d_1, \dots, d_r \in \mathbb{Z}_{>1}$, $0 < q_i < d_i$, $\gcd(d_i, q_i) = 1$, $\alpha := \sum_{i=1}^r \frac{q_i}{d_i} \in \mathbb{Z}$.



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$$\begin{array}{ccc} G \times \mathbb{P}^1 & \xrightarrow{\quad\quad\quad} & \mathbb{Z}^1 \times \mathbb{P}^1 \\ (p, z) & \longmapsto & (\tau(p), z^k). \end{array}$$



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$$(1 \text{ mód } k) \cdot (p, z) \longmapsto (\eta(p), \zeta_k^{-1} z)$$

$$\begin{array}{ccc} & S := (G \times \mathbb{P}^1) / (\mathbb{Z}/k) & \\ \tau_2 \nearrow & & \searrow \\ G \times \mathbb{P}^1 & \xrightarrow{\quad} & \mathbb{Z}^1 \times \mathbb{P}^1 \\ (p, z) \longmapsto & & (\tau(p), z^k). \end{array}$$



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Cubierta cíclica de \mathbb{P}^1

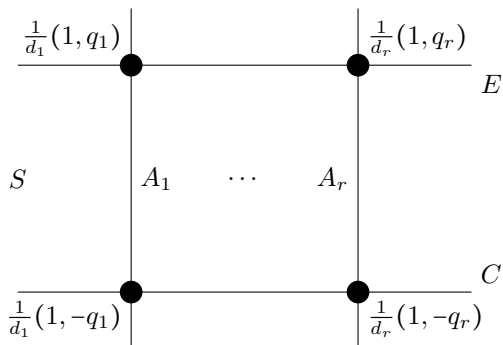
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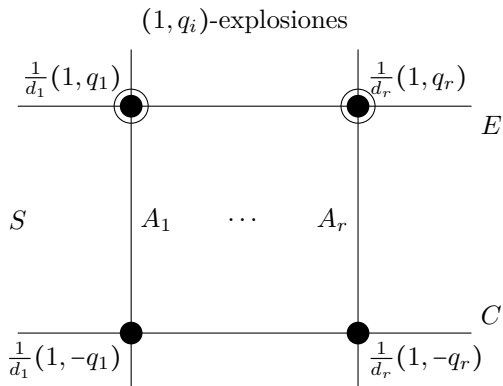
$$\begin{array}{ccc}
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 \mathbb{P}^1 & \xleftarrow{\pi_S} S := (G \times \mathbb{P}^1) / (\mathbb{Z}/k) \xrightarrow{\pi_G} & \mathbb{P}^1 \\
 & \nearrow \tau_2 & \searrow \\
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Construcción birracional de S

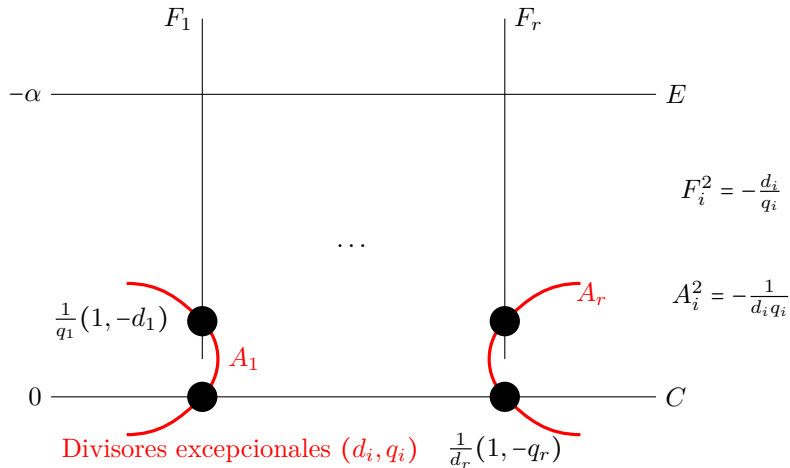


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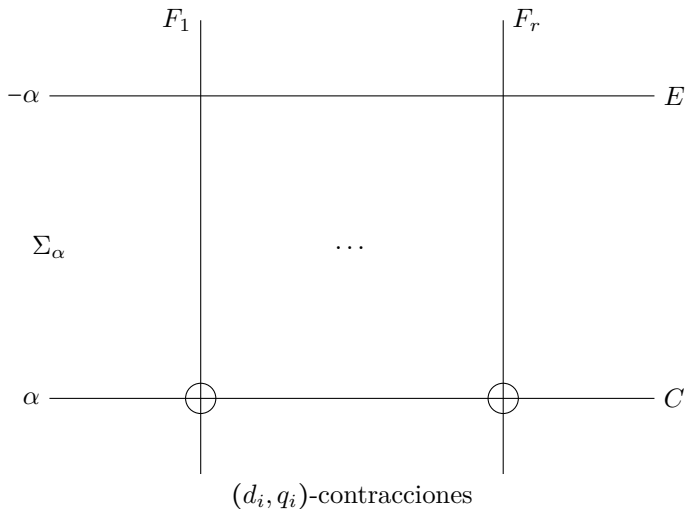


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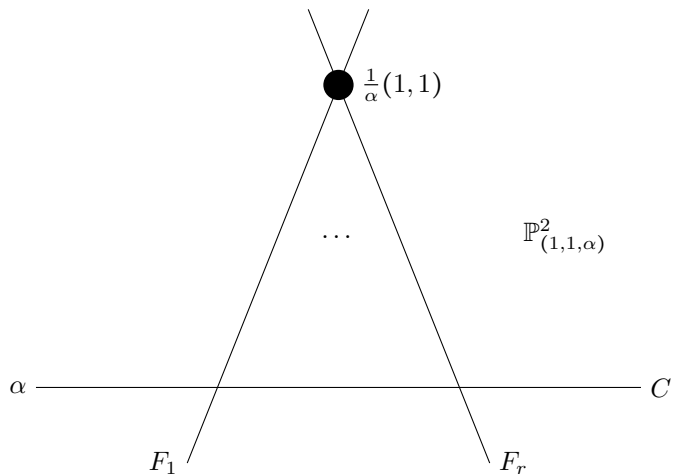
\hat{S}



Construcción birracional de S



Construcción birracional de S



Índice

Grupo de clases y divisor canónico

Proposición

$$\mathrm{Cl}(S) \cong \mathbb{Z}^2 \oplus \bigoplus_{i=1}^h \mathbb{Z}/m_i, \quad 1 \leq h < r, \quad m_{i-1} \mid m_i, \quad m_h = k,$$

$$\prod_{i=1}^h m_i = \frac{d_1 \cdot \dots \cdot d_r}{k} \quad T := E - C \text{ de orden } k.$$



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1. $(d_i) = \overbrace{(k, \dots, k)}^{n \text{ veces}}, \# \mathrm{Tor} \mathrm{Cl}(S) = k^{n-1}.$



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Ejemplos

- $(d_i) = \overbrace{(k, \dots, k)}^{n \text{ veces}}, \# \mathrm{Tor} \mathrm{Cl}(S) = k^{n-1}.$
- (p, q, r) coprimos dos a dos, $(d_i) = (qr, pr, pq), \# \mathrm{Tor} \mathrm{Cl}(S) = pqr = k.$



Representación de las clases en retículo

Dos sucesiones exactas

$$0 \longrightarrow \begin{array}{c} \mathbb{Z}\langle C, E \rangle \\ \text{Cl}_{\mathbb{H}}(S) \\ \mathbb{Z} \oplus \mathbb{Z}/k \end{array} \hookrightarrow \text{Cl}(S) \longrightarrow \mathbb{Z} \oplus \bigoplus_{i=1}^{h-2} \mathbb{Z}/m_i \longrightarrow 0$$



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$$\begin{array}{ccccccc} 0 & \longrightarrow & \begin{array}{c} \mathbb{Z}\langle C, E \rangle \\ \text{Cl}_{\mathbb{H}}(S) \\ \mathbb{Z} \oplus \mathbb{Z}/k \end{array} & \hookrightarrow & \text{Cl}(S) & \longrightarrow & \mathbb{Z} \oplus \bigoplus_{i=1}^{h-2} \mathbb{Z}/m_i \longrightarrow 0 \\ 0 & \longrightarrow & \text{Tor Cl}(S) & \hookrightarrow & \text{Cl}(S) & \longrightarrow & L := \mathbb{Z}\left[\frac{1}{k}\right] \oplus \mathbb{Z} \longrightarrow 0 \\ & & & & D & \longmapsto & \ell_D := \left(\begin{array}{cc} C \cdot D, & F \cdot d \\ \varphi_D & c_D \end{array} \right) \end{array}$$



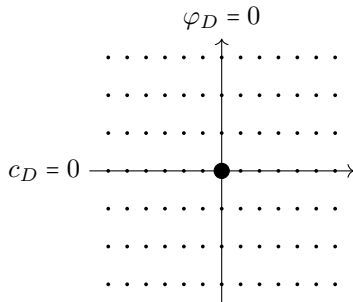
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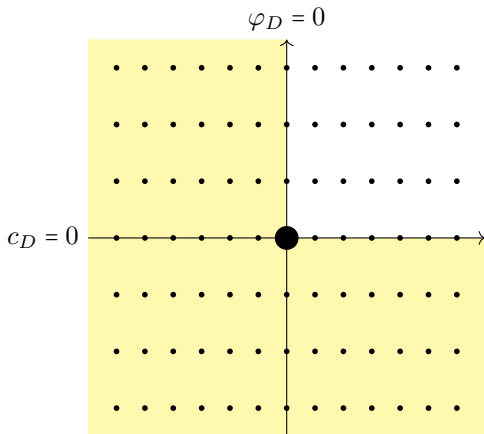
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Índice

Cálculo de H^0



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Hacia $H^0(S; \mathcal{O}(D))$

$$\blacktriangleright D \sim cC + eE + \sum_{i=1}^r a_i A_i + fF \sim c_D C + \sum_{i=1}^r \hat{a}_i A_i + \hat{f}F$$



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Teorema

$$h^0(S, \mathcal{O}_S(D)) = \sum_{j=0}^{c_D} \max\{b_j(D), 0\}$$

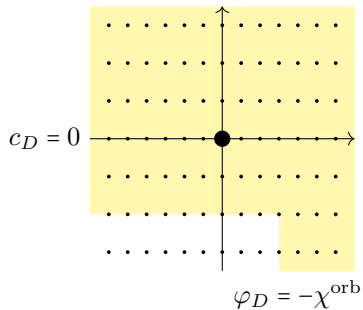


H^1, H^2

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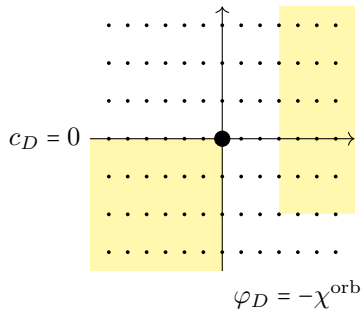
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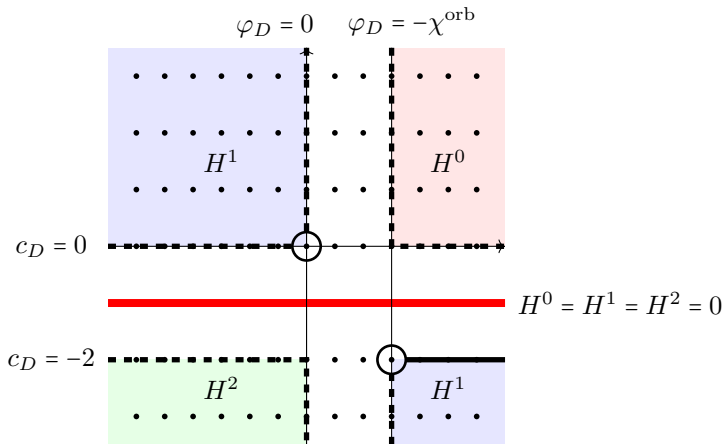
- ▶ $b_j(D) \geq 0$ si $c_D \geq 0$, $\varphi_D > -\chi^{\text{orb}}$

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Cubiertas verticales

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$$D_t : (k\tilde{c}, k\tilde{e}, \overbrace{g_1, \dots, g_1}^{k \text{ veces}}, \dots, \overbrace{g_s, \dots, g_s}^{k \text{ veces}})$$



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$$D_t^i : \left(\frac{k}{d_i} \tilde{c}, \frac{k}{d_i} \tilde{e}, \overbrace{g_1, \dots, g_1}^{\frac{k}{d_i} \text{ veces}}, \dots, \overbrace{g_s, \dots, g_s}^{\frac{k}{d_i} \text{ veces}} \right)$$



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$$\begin{array}{ccccc}
 \pi_t^{-1}(F) & \longrightarrow & \pi_t^{-1}(A_i) & \longrightarrow & Y \\
 \downarrow t:1 & & \downarrow t:1 & & \hat{\pi}_t \downarrow t:1 \\
 F & \longrightarrow & A_i & \longrightarrow & \mathbb{P}^1 \\
 z & \longmapsto & z^{d_i} & \longmapsto & z^k
 \end{array}
 \quad D = (\tilde{c}, \tilde{e}, g_1, \dots, g_s)$$



Cubiertas horizontales

Cubierta asociada a (d, D, H) , $D = cC + eE + \sum_{j=1}^s g_j G_j$

1. $H \sim \gamma C + \eta E + T' \quad D \sim \gamma C + \eta E \quad dT' \sim 0$

2. $T' \in \text{Tor Cl}(S) \quad mT' \in \text{Cl}_H(S) \iff mT' = 0 \quad \text{ord } T' = \frac{d}{t}.$

3. $c = d\gamma + k\tilde{c} \quad e = d\eta + k\tilde{e} \quad \tilde{c} + \tilde{e} + \sum_{j=1}^s g_j = 0$

4. $n := \gcd(d, c, e, g_j) = \gcd(d, k\tilde{c}, k\tilde{e}, g_j) = \gcd(d, k\tilde{e}, g_j).$

5. $(\pi_n)_| : \pi_n^{-1}(E) \longrightarrow E$ cubierta intermedia de orden n .

6. $(\hat{\pi}_t)_| : Y \longrightarrow \mathbb{P}^1$ máximo común divisor de cubiertas sobre fibras

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Teorema

$$\Delta_{\pi}^{HEV} = \Delta_{(\pi_n)_|}^{HEV} \cdot \Delta_{(\hat{\pi}_t)_|}^{HEV}$$

Caso general

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$$D'' = \sum_{j \notin J_2} m_j D_j \quad H'' = \frac{d}{d''} H - \frac{1}{d''} \sum_{j \in J_2} m_j D_j$$

- ▶ $D_j \cdot C = 0 \iff D_j = C, E, G.$

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- ▶ $J_2 = \{j \in J \mid D_j \cdot C \neq 0\}$ $L^{(l)} \cdot C = 0 \iff d$ divide a $l m_j, \forall j \in J_2.$
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- ▶ Cubierta horizontal π'' (d'', D'', H'')

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Teorema

$$\Delta_{\pi}^{HEV} = \frac{\Delta_{\pi'}^{HEV} \Delta_{\pi''}^{HEV}}{\Delta_{\pi'''}^{HEV}}$$





¡¡Gracias por aguantar
hasta aquí!!