

Plane octic curves with $6\mathbb{E}_6$ singular points and non hyperelliptic Riemann surfaces of genus 3

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Abstract Riemann surfaces and embedding properties of plane curves?????

- Properties of $(\mathbb{P}^2, \mathcal{C}^{\text{alg}}) / \mathbb{C}$.



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- ▶ $D \in \text{Div}(\mathcal{C})$, $V \subset H^0(\mathcal{C}; \mathcal{O}_{\mathcal{C}}(D))$, $\dim_{\mathbb{C}} V = 3$.



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Abstract Riemann surfaces and embedding properties of plane curves?????

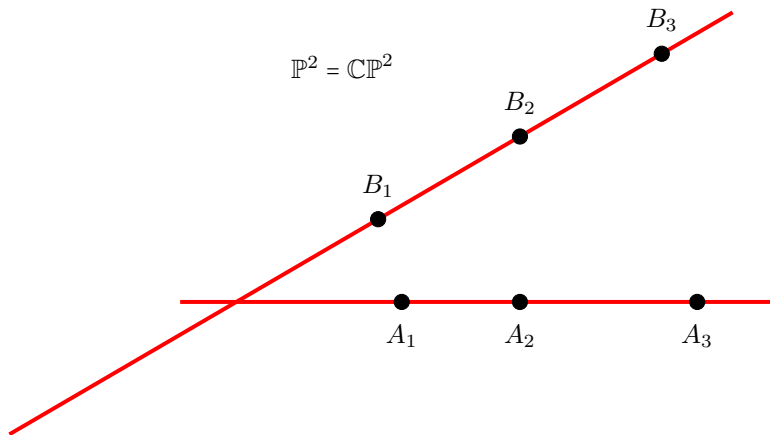
- ▶ Properties of $(\mathbb{P}^2, \mathcal{C}^{\text{alg}}) / \mathbb{C}$.
- ▶ Abstract Riemann surface $\mathcal{C}$.
- ▶ $D \in \text{Div}(\mathcal{C})$, $V \subset H^0(\mathcal{C}; \mathcal{O}_{\mathcal{C}}(D))$, $\dim_{\mathbb{C}} V = 3$.
- ▶ *Intrinsic embedding/map:*

$$\begin{array}{ccc} \mathcal{C} & \hookrightarrow & \mathbb{P}(H^0(\mathcal{C}; \mathcal{O}_{\mathcal{C}}(D))^{\vee}) \\ & \searrow & \downarrow \\ & & \mathbb{P}(V^{\vee}) \end{array}$$



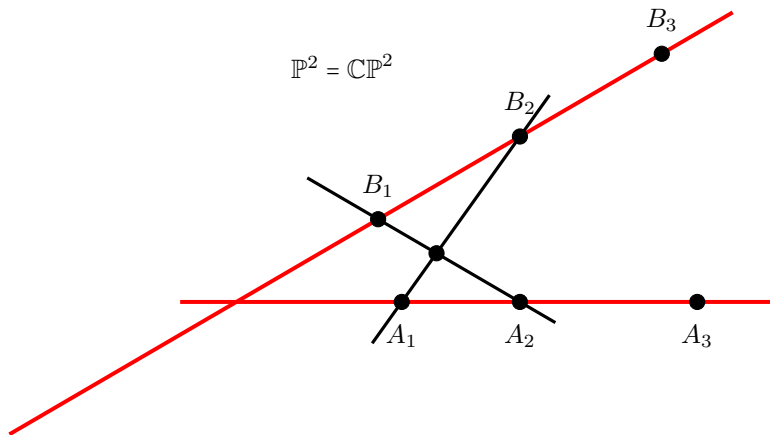
Symplectic curves

Algebraic and symplectic Pappus arrangements



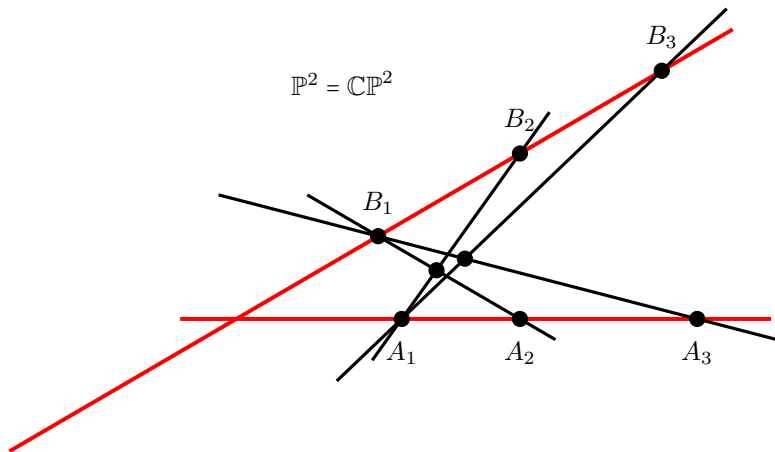
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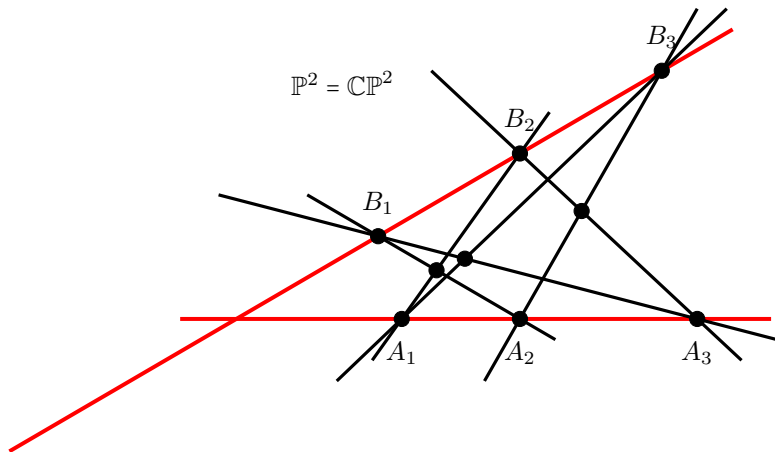
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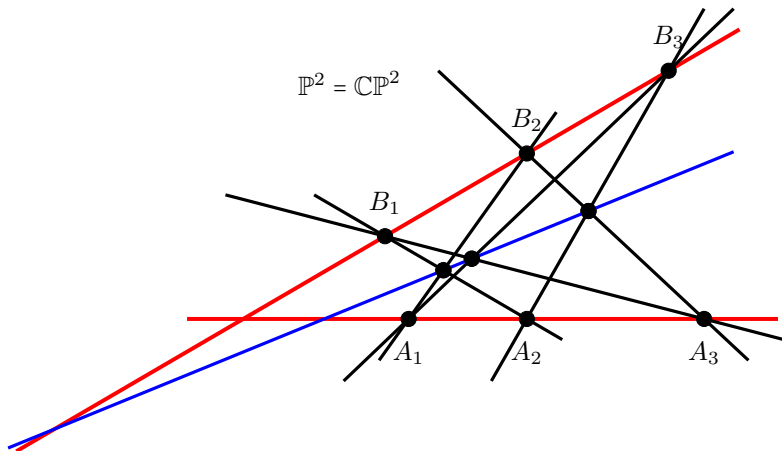
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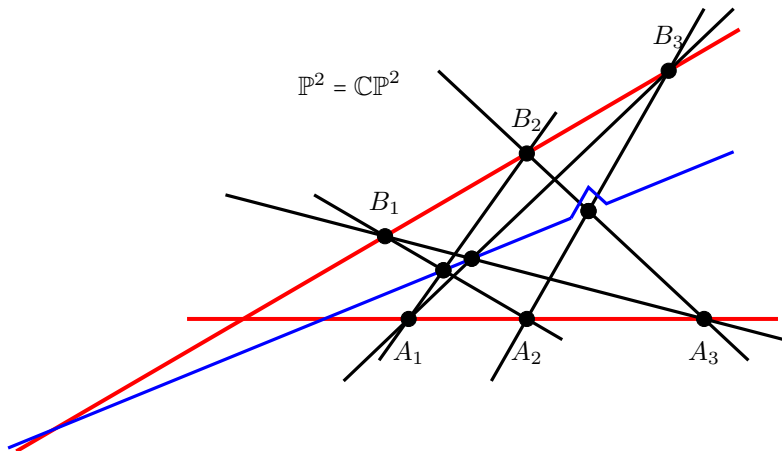
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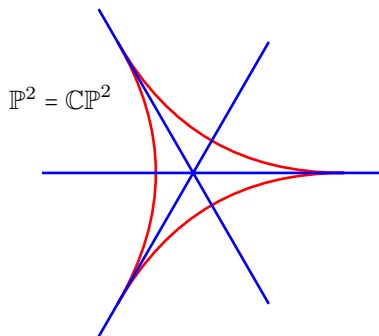
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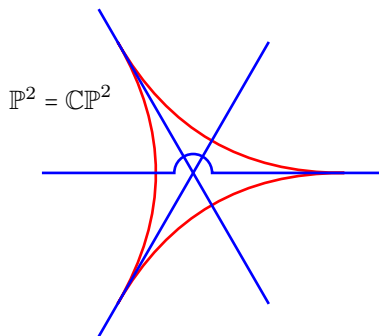
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Algebraic and symplectic tangent lines to the cusps of the deltoid



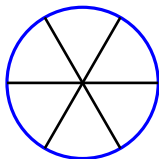
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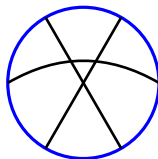


Symplectic curves

Algebraic versus Symplectic: Topology



- ▶ Cone over 3-component Hopf link.
- ▶ $\pi_1^{\text{loc}} = \langle x_1, x_2, x_3 \mid x_1 x_2 x_3 \text{ central} \rangle$.
- ▶ $\pi_1^{\text{loc}} \cong \mathbb{F}_2 \times \mathbb{Z}$.

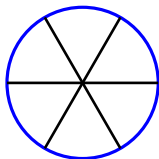


- ▶ Complement of 3 generic lines in $\mathbb{C}^2$.
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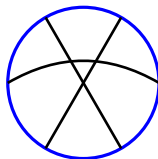


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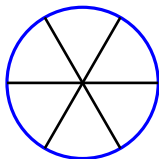
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Zariski-van Kampen

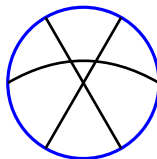
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Zariski-van Kampen

Fundamental group of the complement of an algebraic curve

Seifert-van Kampen

Fundamental group of the complement of a symplectic deformation of an algebraic curve

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 - ▶ Rational octic with 3 singular points homeomorphic to $y(x^3 - y^5) = 0$.
 - ▶ Cremona transformation of the symplectic deltoid.
 - ▶ Non-isotopic since no such algebraic curve exists.
- ▶ Problem: find *combinatorics* (degree and topological type of singularities) of algebraic curves having symplectic representatives non-isotopic to the algebraic ones.



Symplectic Kummer cover

Kummer covers

- ▶ Algebraic cover: $(\mathbb{Z}/2)^2$ -Galois cover ramified along $xyz = 0$

$$\begin{aligned}\mathbb{P}^2 &\longrightarrow \mathbb{P}^2 \\ [x : y : z] &\longmapsto [x^2 : y^2 : z^2]\end{aligned}$$



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Kummer covers

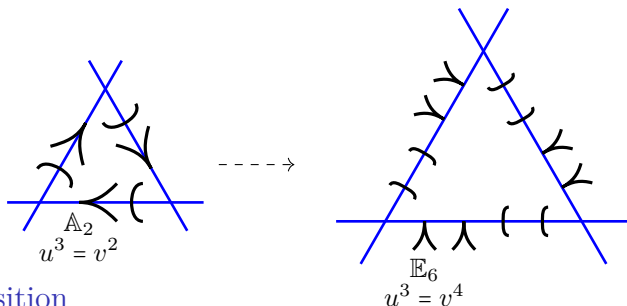
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- ▶ Symplectic cover: $(\mathbb{Z}/2)^2$ -Galois symplectic cover ramified along three generic symplectic lines.



Symplectic Kummer cover



Proposition

There exists a symplectic curve $\mathcal{C}^{\text{symp}}$ of degree 8 (and genus 3) with 6 singular points of type $\mathbb{E}_6$, $G^{\text{symp}} := \pi_1(\mathbb{P}^2 \setminus \mathcal{C}^{\text{symp}})$,

$$G^{\text{symp}} = \left\langle c_1, \dots, c_4 \left| \begin{array}{l} [c_i, c_j] = 1 \quad , \quad c_i c_j c_i = c_j c_i c_j \quad , \quad (c_2 c_1 c_3 c_4)^2 = 1 \\ (i,j)=(1,3),(2,4) \quad (i,j)=(1,2),(2,3),(3,4) \end{array} \right. \right\rangle$$



Algebraic octic curves with $6\mathbb{E}_6$

Strategy

- ▶ Consider $F_8(x, y, z) := \sum_{i+j+k=8} a_{i,j,k} x^i y^j z^k$.



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- ▶ System of equations in $\mathbb{C}[\{a_{i,j,k} \mid i+j+k=8\} \cup \{x_0, y_0, x_1, y_1\}]$ (containing many degenerate solutions).



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- ▶ System of equations in $\mathbb{C}[\{a_{i,j,k} \mid i+j+k=8\} \cup \{x_0, y_0, x_1, y_1\}]$ (containing many degenerate solutions).
- ▶ Complete failure!
- ▶ Try symmetric curves!



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Theorem

$\mathcal{C} \subset \mathbb{P}^2$, $\deg \mathcal{C} = 8$, $\text{Sing } \mathcal{C} = \{6\mathbb{E}_6\}$, $\text{Aut}(\mathbb{P}^2, \mathcal{C}) \neq \{1\}$. Then,
 $\text{Aut}(\mathbb{P}^2, \mathcal{C}) \cong \mathbb{Z}/2, \mathbb{Z}/3$ and $\mathcal{C}$ is projectively equivalent to:

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Topology

$(\mathbb{P}^2, \mathcal{C}^{\text{symp}l}) \not\cong (\mathbb{P}^2, \mathcal{C}_2)$, $(\mathbb{P}^2, \mathcal{C}^{\text{symp}l}) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$, $(\mathbb{P}^2, \mathcal{C}_2) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$,

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$\mathcal{C} \subset \mathbb{P}^2$, $\deg \mathcal{C} = 8$, $\text{Sing } \mathcal{C} = \{6\mathbb{E}_6\}$, $\text{Aut}(\mathbb{P}^2, \mathcal{C}) \neq \{1\}$. Then,
 $\text{Aut}(\mathbb{P}^2, \mathcal{C}) \cong \mathbb{Z}/2, \mathbb{Z}/3$ and $\mathcal{C}$ is projectively equivalent to:

$\mathbb{Z}/2$. $\mathcal{C}_2 := \{F_2(x, y, z^2) = 0\}$, $G_2 := \pi_1(\mathbb{P}^2 \setminus \mathcal{C}_2)$, $G_{(n)} := G^{(n)}/G^{(n+1)}$

$$\begin{array}{llll} F_2 \in \mathbb{Q}[x, y, z] & G_{2(0)} \cong \mathbb{Z}/8 & G_{2(1)} \cong \mathbb{Z}/3 & G_{2(2)} \cong (\mathbb{Z}/2)^4 \\ & G_{(0)}^{\text{symp}l} \cong \mathbb{Z}/8 & G_{(1)}^{\text{symp}l} \cong \mathbb{Z}/3 & G_{(2)}^{\text{symp}l} \cong (\mathbb{Z}/2)^6 \end{array}$$

$\mathbb{Z}/3$. $\mathcal{C}_{3,i} := \{F_{3,i}(x, y, z) = 0\}$, $\pi_1(\mathbb{P}^2 \setminus \mathcal{C}_{3,i}) \cong \mathbb{Z}/8$,

- ▶ $F_3 \in \mathbb{K}[x, y, z]$,
- ▶ $\mathbb{K} = \mathbb{Q}[t]/(t^4 - 2t^3 + t^2 - 2t - 2)$
- ▶ $\eta_1, \dots, \eta_4$ roots of p , $\eta_1, \eta_2 \in \mathbb{R}$, $\bar{\eta}_3 = \eta_4$.
- ▶ $F_{3,i} = F(t = \eta_i)$.

Topology

$(\mathbb{P}^2, \mathcal{C}^{\text{symp}l}) \not\cong (\mathbb{P}^2, \mathcal{C}_2)$, $(\mathbb{P}^2, \mathcal{C}^{\text{symp}l}) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$, $(\mathbb{P}^2, \mathcal{C}_2) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$,
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$(\mathbb{P}^2, \mathcal{C}^{\text{symp}^1}) \not\cong (\mathbb{P}^2, \mathcal{C}_2)$, $(\mathbb{P}^2, \mathcal{C}^{\text{symp}^1}) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$, $(\mathbb{P}^2, \mathcal{C}_2) \not\cong (\mathbb{P}^2, \mathcal{C}_{3,i})$,
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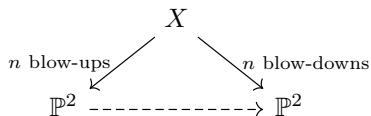
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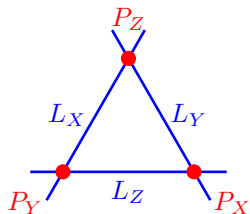
Cremona transformations

General idea



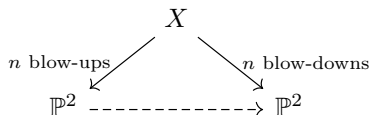
Easy way: n smooth rational pairwise disjoint (-1) -curves in X

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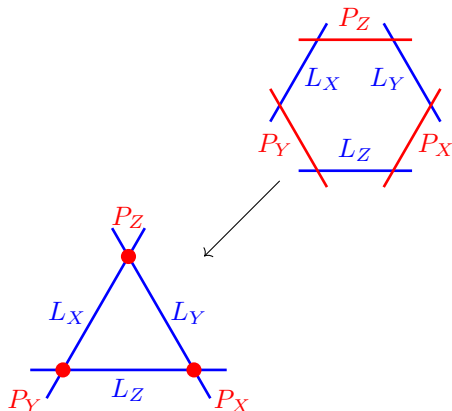


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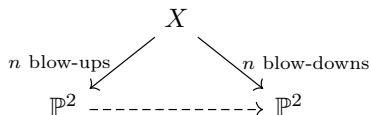


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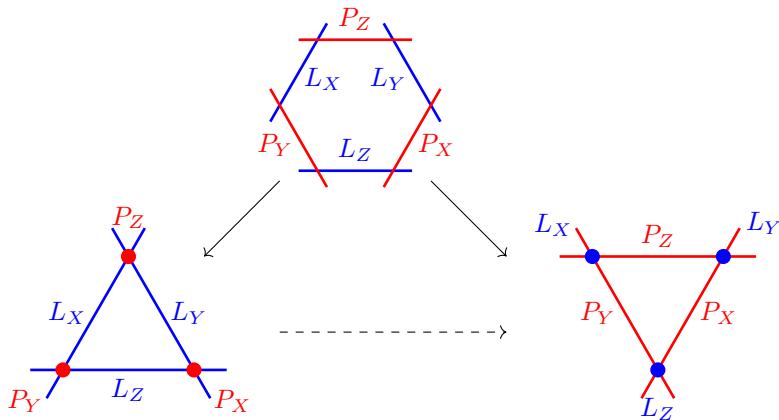


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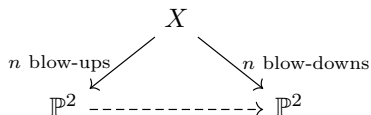


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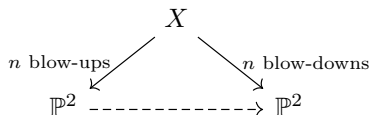
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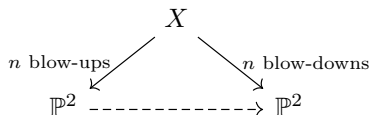
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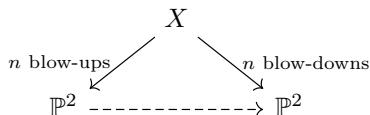
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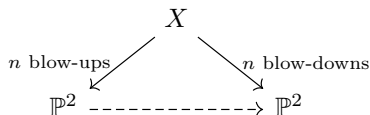
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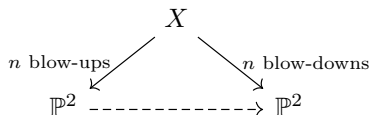
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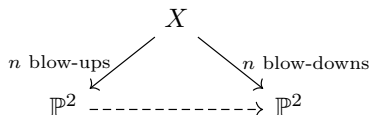
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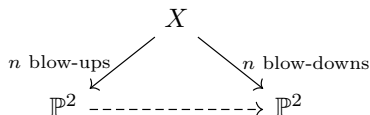
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Cremona transformations

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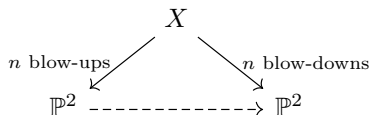
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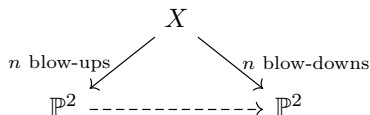
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- ▶ $Q_1, \dots, Q_6 \subset X$ smooth rational pairwise disjoint (-1) -curves.
- ▶ $\sigma: X \rightarrow \mathbb{P}^2$, contraction of $Q_1, \dots, Q_6$: $P_1, \dots, P_6$ conics.



Cremona transformations

General idea



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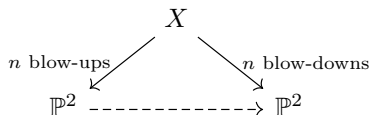
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- ▶ $\gamma := \sigma \circ \rho^{-1}: \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$



Cremona transformations

General idea



Easy way: n smooth rational pairwise disjoint (-1) -curves in X

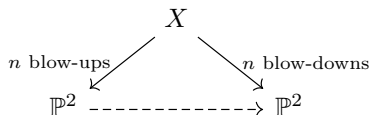
Proposition

$\mathcal{C} \subset \mathbb{P}^2$, $\deg \mathcal{C} = 8$, $\text{Sing } \mathcal{C} = \{6\mathbb{E}_6\}$, $\tilde{\mathcal{C}} := \gamma(\mathcal{C})$ is a smooth quartic curve,
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Cremona transformations

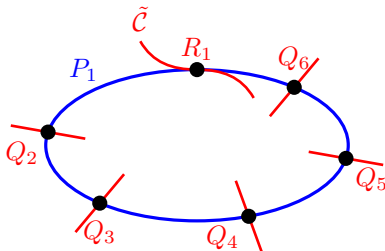
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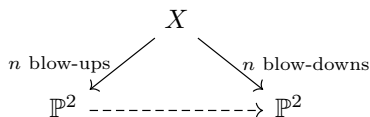
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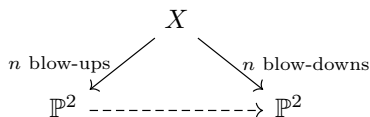
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Consequences

- ▶ $\tilde{\mathcal{C}}$ is the normalization of $\mathcal{C}$: non hyperelliptic Riemann surface of genus 3.

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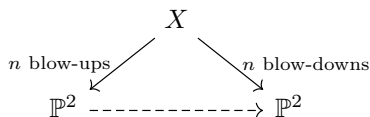
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- ▶ $\tilde{\mathcal{C}} \hookrightarrow \mathbb{P}(H^0(\tilde{\mathcal{C}}; \mathcal{K}_{\tilde{\mathcal{C}}})^\vee)$: canonical!



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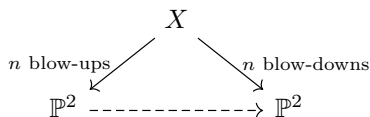
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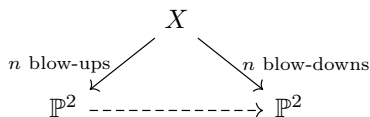
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- ▶ $Q := Q_1 + \cdots + Q_6$: $Q - Q_i + 3R_i \in |\mathcal{K}_{\tilde{\mathcal{C}}}^{\otimes 2}|$
- ▶ Can one find all $(\mathcal{C}, (Q_1, \dots, Q_6), (R_1, \dots, R_6))$ satisfying the above conditions?

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Easy way: n smooth rational pairwise disjoint (-1) -curves in X

Proposition

$\mathcal{C} \subset \mathbb{P}^2$, $\deg \mathcal{C} = 8$, $\text{Sing } \mathcal{C} = \{6\mathbb{E}_6\}$, $\tilde{\mathcal{C}} := \gamma(\mathcal{C})$ is a smooth quartic curve, $\tilde{\mathcal{C}} \cdot P_i = Q_1 + \cdots + \widehat{Q_i} + \cdots + Q_6 + 3R_i$.

Consequences

- ▶ $\tilde{\mathcal{C}}$ is the normalization of $\mathcal{C}$: non hyperelliptic Riemann surface of genus 3.
- ▶ $\tilde{\mathcal{C}} \hookrightarrow \mathbb{P}(H^0(\tilde{\mathcal{C}}; \mathcal{K}_{\tilde{\mathcal{C}}})^\vee)$: canonical!
- ▶ $Q := Q_1 + \cdots + Q_6$: $Q - Q_i + 3R_i \in |\mathcal{K}_{\tilde{\mathcal{C}}}^{\otimes 2}|$
- ▶ Can one find all $(\mathcal{C}, (Q_1, \dots, Q_6), (R_1, \dots, R_6))$ satisfying the above conditions?
- ▶ Most probably discrete solutions.



Gracias por su atención