Braid monodromy and topology of complexified real arrangements [1]

Enrique Artal (Universidad de Zaragoza)

Sapporo, September 2003

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Topology of $\mathscr{A} \otimes_{\mathbb{R}} \mathbb{C}$ depends on properties of $\mathscr{A}$.

Theorem 1. *There exist two arrangements of lines* $\mathscr{A}_1, \mathscr{A}_2$ *in* $\mathbb{RP}^2$ *such that* $\mathscr{L}(\mathscr{A}_1) = \mathscr{L}(\mathscr{A}_2)$ *and their complexifications* $\mathscr{A}_i^{\mathbb{C}} := \mathscr{A}_i \otimes_{\mathbb{R}} \mathbb{C}$, $i = 1, 2$, *have non-homeomorphic embeddings in* $\mathbb{CP}^2$.

Theorem 1. *There exists an arrangement of lines* $\mathscr{A}$ *in* $\mathbb{Q}(\sqrt{5})\mathbb{P}^2$ *such that the complexifications* $\mathscr{A}_1^{\mathbb{C}}, \mathscr{A}_2^{\mathbb{C}}$ *induced by the two inclusions* $\mathbb{Q}(\sqrt{5}) \hookrightarrow \mathbb{C}$ *have non-homeomorphic embeddings in* $\mathbb{CP}^2$.

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Their profinite completions are isomorphic.

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$\Sigma^{\text{ord}}(\mathscr{L})$ and $\mathscr{M}^{\text{ord}}(\mathscr{L})$ for ordered $\mathscr{L}$

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Rybnikov's made use of McLane arrangement in $\mathbb{F}_3\mathbb{P}^2$.

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- Each M_i contains one double point $\{M_i, L_i\}$

Properties of $\operatorname{Aut}\mathscr{L}$:

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Drop one M_i: Falk-Sturmfels combinatorics [3]

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Remark 6. Braid monodromy, monodromy group, pseudo-Coxeter element well defined up to the action of the same inner automorphisms of $\mathbb{P}_n$. Each string is attached to one line. If $\mathscr{H}$ is ordered, strings are compatible with the order.

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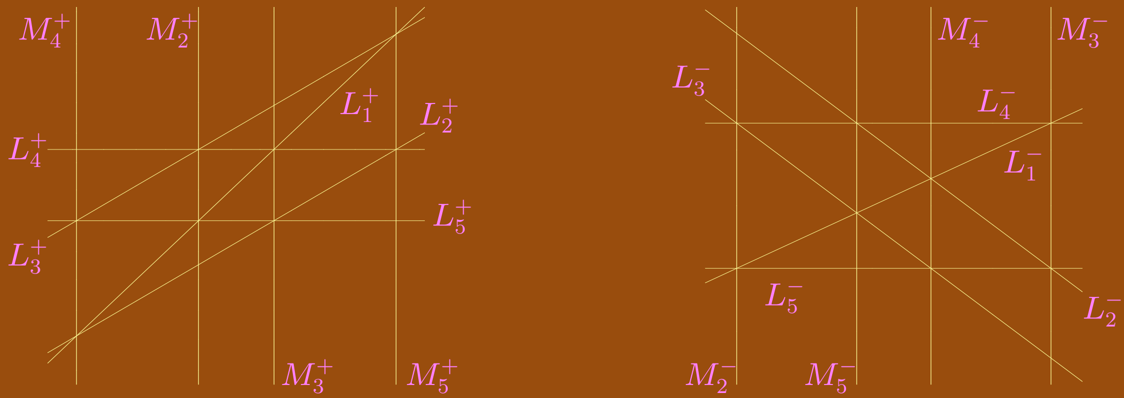
line-order and plane-line-orientation-preserving homeomorphism.
Choose $L_\infty \in \mathscr{C}_i^v$ *(same ordered line),* P_i *point at infinity of vertical lines.*
Then monodromy groups and pseudo-Coxeter elements of affine $\mathscr{C}_1^h$ *and* $\mathscr{C}_2^h$ *are conjugate by the same element in* $\mathbb{P}_m$, $m = \#\mathscr{C}_1^h = \#\mathscr{C}_2^h$.

6. Topology of the realizations

$\mathscr{L}^{\pm}$ representing elements of $\mathscr{M}^{\text{ord}}(\mathscr{L})$, see Proposition 3(c), by the choice of $\gamma^{\pm} := \frac{-1\pm\sqrt{5}}{2}$.



(a) Arrangement $\mathscr{L}^+$ (b) Arrangement $\mathscr{L}^-$

Figure 1: Affine arrangements

Take the horizontal part of $\mathscr{L}^+$ and consider $D^+ \subset \mathbb{C}$.

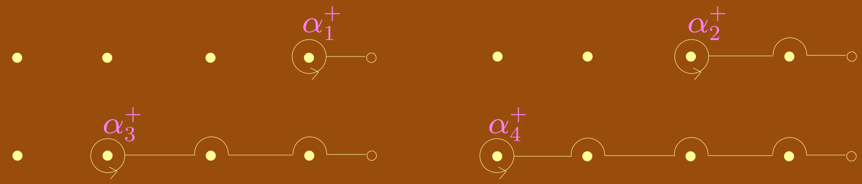


Figure 2: Free generators of $\pi_1(\mathbb{C} \setminus D^+; \circ)$

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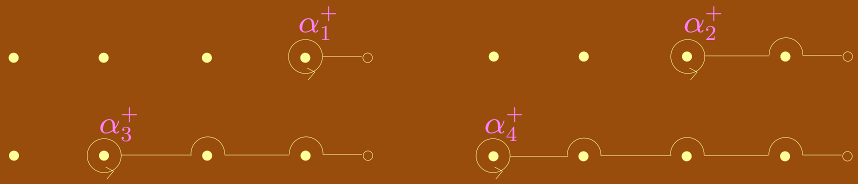

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Proposition 8. *The braid monodromy of the affine part of $\mathscr{L}^+$ with respect to vertical projection of figure 1(a) is given by:*

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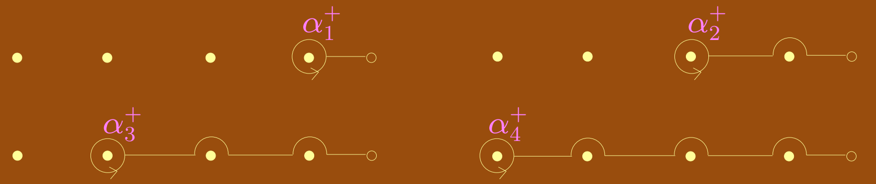

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$$
\begin{aligned}
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&= \\
&= \\
&=
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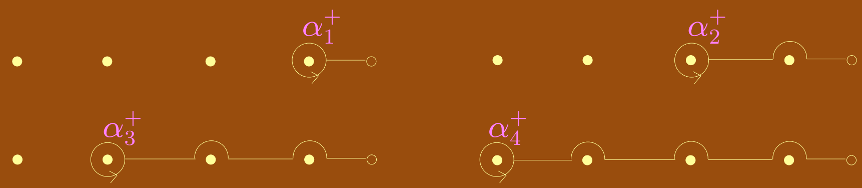

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$$\rho^+(\alpha_1^+) = \sigma_1^2 \sigma_3^2$$
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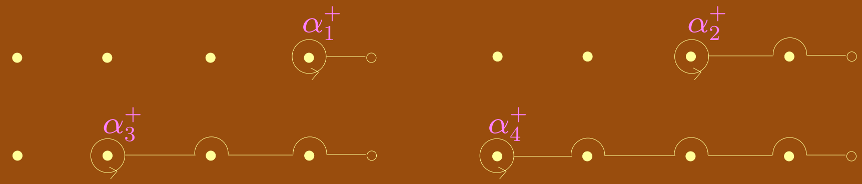

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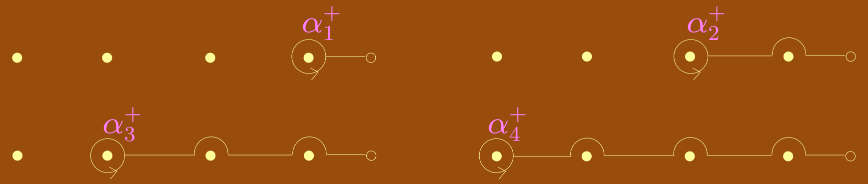

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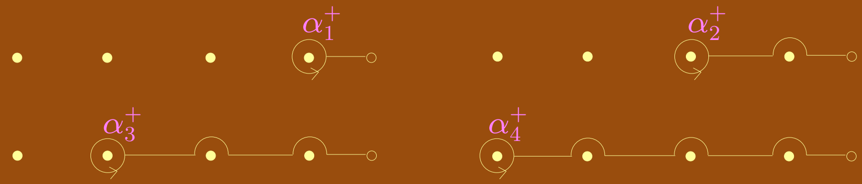

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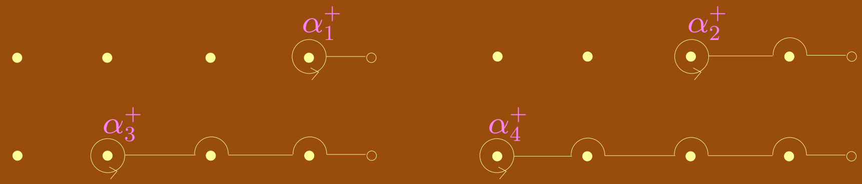

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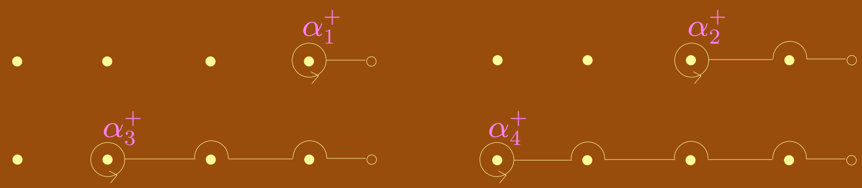

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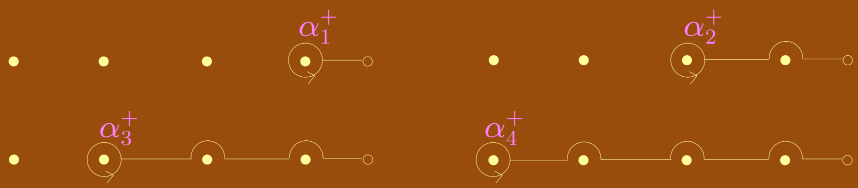

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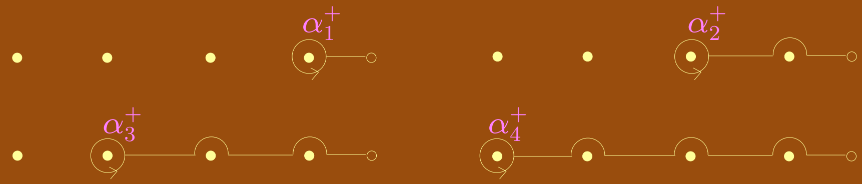

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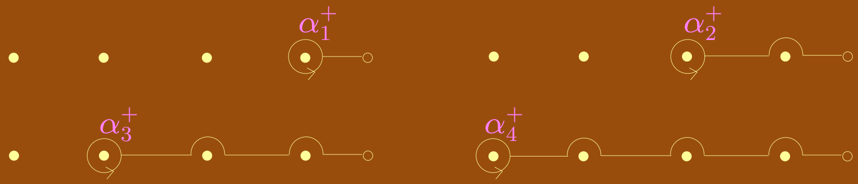

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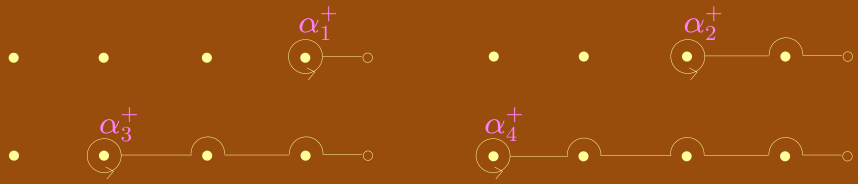

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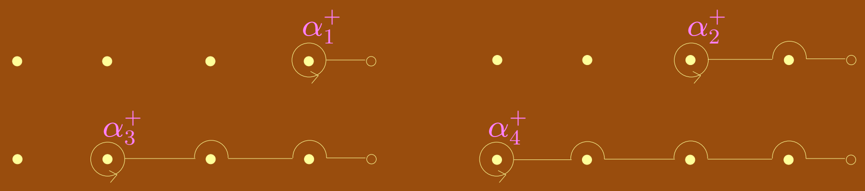

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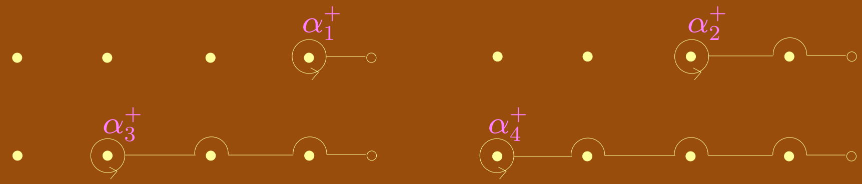

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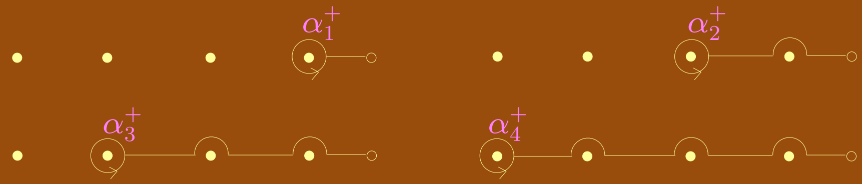

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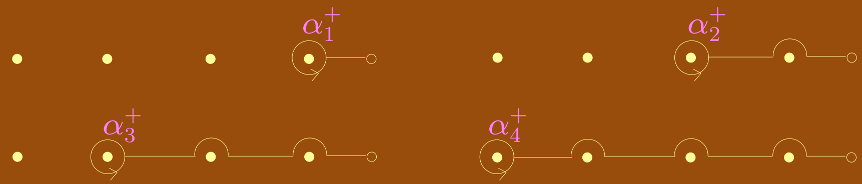

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The strings correspond to the lines $(L_1^+, L_3^+, L_2^+, L_4^+, L_5^+)$.

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The strings correspond to the same order if we consider $\rho^- := \tau^{-1}\tilde{\rho}^-\tau$,

$$\tau := \sigma_3\sigma_4\sigma_2\sigma_3\sigma_2 \in \mathbb{B}_5$$

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$$h : (\mathbb{P}^2, \bigcup \mathscr{L}^+) \to (\mathbb{P}^2, \bigcup \mathscr{L}^-)$$

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$$\mathbb{P}_5 \xrightarrow{\text{Burau}} \mathrm{GL}(5; \mathbb{Z}[t, t^{-1}]) \to \mathrm{GL}(5; \mathbb{F}_5).$$

where $t \mapsto 2 \mod 5$. Let $G := \mathrm{Im}\,\beta$, $\#G = 58032 \cdot 10^6$, $H^\pm := \beta(K^\pm)$, $\#H^\pm = 30000$, $d^\pm := \beta(c^\pm)$.

Then, using a GAP4 [4] program we show that $\nexists x \in G$ such that $H^+ = x^{-1}H^-x$ and $d^+ = x^{-1}d^-x$. Apply proposition 7.

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GAP4 programs can be found at

`http://riemann.unizar.es/geotop/pub/gap-programs/lines/monodromy_groups.`

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$$N^{\pm} : \gamma^{\pm} x + (\gamma^{\pm} + 1) y + z = 0$$

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$\mathscr{A}$ combinatorics of $\mathscr{H}^{\pm}$

Proposition 11. *The automorphism group of $\mathscr{A}$ is trivial and there is no homeomorphism between the pairs $(\mathbb{P}^2, \bigcup \mathscr{A}^{+})$ and $(\mathbb{P}^2, \bigcup \mathscr{A}^{-})$.*

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