$$Z_f = \frac{1}{Ns+1} + \alpha(N,s)\, Z_g\left(s+\frac{1}{N}\right) - \frac{s}{s+1} \sum_{e/N,\, e\neq 1} \frac{e^2-1}{N} Z_g^{[e]}\left(s+\frac{1}{N}\right)$$

$$Z_g^{(e)} = \sum_{\ell \in A_e} Z_g^{[\ell]}$$

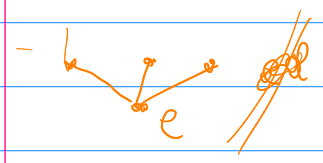
$$Z_g^{[e]} = \sum_{\ell \in A_e} a_{e\ell}\, Z_g^{(\ell)}$$

$$A_e = \left\{ \ell \;\middle|\; e \,/\, \ell/N \right\} \qquad Z_g^{[e]} = Z_g^{(e)} - \sum_{j=1}^{r} \sum_{\ell \in A_{e,j}} Z_g^{[\ell]} =$$

$$= Z_g^{e} - \sum_{\ell \in A_{e,1}} Z_g^{[\ell]} - \sum_{j=2}^{r} \sum_{\ell \in A_{e,j}} Z_g^{[\ell]} =$$

$$= Z_g^{e} - \sum_{\ell \in A_{e,1}} Z_g^{(\ell)} + \sum_{\ell \in A_{e,1}} \sum_{j=2}^{r'} \sum_{\ell' \in A_{\ell,j-1}} Z_g^{[\ell']} -$$

$$- \sum_{j=2}^{r} \sum_{\ell \in A_{e,j}} Z_p^{[\ell]}$$



$$A_e = \bigcup_{j=0}^{r} A_{e,j}$$

$$A_{e,e} = \{ e \cdot j \text{ primo} \}$$

$$g(n) = Z_{top}^{\left(\frac{N}{n}\right)} \qquad n \quad n/N$$

$$f(n) = Z_{top}^{\left[\frac{N}{n}\right]}$$

$$g(n) = Z_{top}^{\left(\frac{N}{n}\right)} = \sum_{\frac{N}{n}/\ell/N} Z_{top}^{[\ell]} = \sum_{\tilde{\ell}/n} Z_{top}^{\left[\frac{N}{\tilde{\ell}}\right]} =$$

$$\tilde{\ell} = \frac{N}{\ell}$$

$$= \sum_{\tilde{\ell}/n} f(\tilde{\ell})$$

$$f(n) = \sum_{\ell / n} \mu(\ell)\, g\left(\frac{n}{\ell}\right)$$

$$Z_{top}^{[e]} = f\left(\frac{N}{e}\right) = \sum_{\tilde{\ell} / \frac{N}{e}} \mu(\tilde{\ell})\, g\left(\frac{N}{e\tilde{\ell}}\right) \overset{\ell = \tilde{\ell} e}{=}$$

$$= \sum_{e/\ell/N} \mu\left(\frac{\ell}{e}\right) g\left(\frac{N}{\ell}\right) = \sum_{e/\ell/N} \mu\left(\frac{\ell}{e}\right) Z_{top}^{(\ell)}$$

$Z_{top}^{(e)}(F)$ $\quad e/N$

① Estratificación

② Para cada estrato i $\longleftrightarrow$ $\begin{pmatrix} 0 \\ 0 \\ N \end{pmatrix}, \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$, aristas σ

$\hookrightarrow \operatorname{lcm}\left(N, \gcd(u_i, u_j)\right) = h_i$

$\hookrightarrow e_i^*$ $\qquad e_i = \gcd(N, u_i, u_j)$

$N \quad \gcd(u_i, u_j)$ $\qquad N \gcd(u_i, u_j) = e_i h_i$

$\tau_1 \longrightarrow$ siempre col. como $\dfrac{\frac{1}{Ns+1} - Z_g\left(s + \frac{1}{N}\right)}{Ns+1}$

$\tau_0 \longrightarrow Z_{top}^{(e)}\left(s + \frac{1}{N}\right)$

$\sigma \longrightarrow$ estrato e_i $\quad \dfrac{e_i^2}{N \overline{\Pi}_i (\,)} \rightsquigarrow \dfrac{1}{N} \sum_{\ell' / \ell \neq \ell / N} Z_{top}^{[\ell]}\left(s + \frac{1}{N}\right)$

$$Z_{top}(F) \simeq A_{\tau_0} + B_{\tau_1} \frac{-s}{s+1} C_\sigma$$

$$e/N \quad Z^{(e)}_{top}(F) = A_{\tau_0} + B^{(e)}_{\tau_1} - C_\sigma$$

$$B_{\tau_1} = Z_{top}\left(\beta, s+\frac{1}{N}\right) \qquad B^{(e)}_{\tau_1} = Z^{(e)}_{top}\left(\beta, s+\frac{1}{N}\right)$$

$$(Ns+1) A_{\tau_0} = 1 + Z_{top}\left(\beta, s+\frac{1}{n}\right)$$

$$C_\sigma = \sum_{1 \neq k/N} \frac{k^2-1}{N} Z^{[k]}_{top}\left(\beta, s+\frac{1}{N}\right) = \sum_{1 \neq \ell/N} \frac{\ell^2}{N} Z^{[\ell]}_{top}\left(\beta, s+\frac{1}{N}\right)$$

$$- \frac{1}{N}\left(Z_{top}\left(\beta, s+\frac{1}{N}\right) - Z^{[1]}_{top}\left(\beta, s+\frac{1}{N}\right)\right)$$

$$Z^{[\ell]}_{top}(\beta) = \sum_{\ell/\ell'/N} \mu\left(\frac{\ell'}{\ell}\right) Z^{(\ell')}_{top}(\beta, s)$$

$$\ell_1 \left(\sum_{1 \neq \ell_2/\ell_1/N} \frac{\ell_2^2}{N} \mu\left(\frac{\ell_1}{\ell_2}\right) \right) Z^{(\ell_1)}_{top}\left(\beta, s+\frac{1}{N}\right)$$

$$\sum_{1 \neq \ell_2/\ell_1} \frac{\ell_2^2}{N} \mu\left(\frac{\ell_1}{\ell_2}\right) = h(\ell_1)$$

$$\frac{k(\ell_2)}{\emptyset}$$

$$\frac{1}{N} \sum_{1/\ell_2/\ell_1} \ell_2^2$$

$$h(\ell_1) = \sum_{1=\ell_2/\ell_1} k(\ell_2)\, \mu\left(\frac{\ell_1}{\ell_2}\right)$$

$$k(\ell_1) = \sum_{1/\ell_2/\ell_1} h(\ell_2)$$