

# MORITA THEORY AND ITS GENERALIZATIONS

A GRADUATE COURSE BY DR. OLEG SMIRNOV, COLLEGE OF CHARLESTON

*“Morita’s theorem on equivalence is probably one of the most frequently used single results in modern algebra”*

Arhangelskii, Goodearl, and Huisgen-Zimmermann in *Kiiti Morita 1915–1995*,  
44 (6), AMS Notices, 1997

This course provides an introduction to the classical Morita theory and to some of its more recent versions. The origin of this theory was given by Kiiti Morita in 1958 [5]. It was popularized by Hymen Bass who also brought the theory to its current form [2].

Two generalizations of this theory will be presented as well. The first is a generalization of Morita theory for idempotent algebras developed by Parvathi, Ramakrishna Rao [6] and Garcia, Simon [4]. The second is a version of the theory for torsionfree algebras. It was pioneered by Bruno Müller. I continued his work in [7]. Both of these versions are suitable for unital and some classes of nonunital algebras and provide useful tools for some classes of nonassociative algebras, such as Jordan algebras, structurable algebras, and Kantor pairs.

Every version of Morita theory introduces a concept of equivalence on a certain class of algebras, usually called Morita equivalence, which is related to the equivalence of certain categories of modules. Also the theory gives an algebraic description of such equivalences using Morita contexts. One of the most useful outcomes of the theory is that Morita equivalent algebras share many important properties.

This is a one semester graduate course based on a mini-course I gave at the College of Charleston. The intended audience are students working towards their master or PhD degree. A necessary introduction to the category theory with the emphasis on the category of modules will be given at the beginning of the course.

## REFERENCES

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